HUBBLE'S LAW
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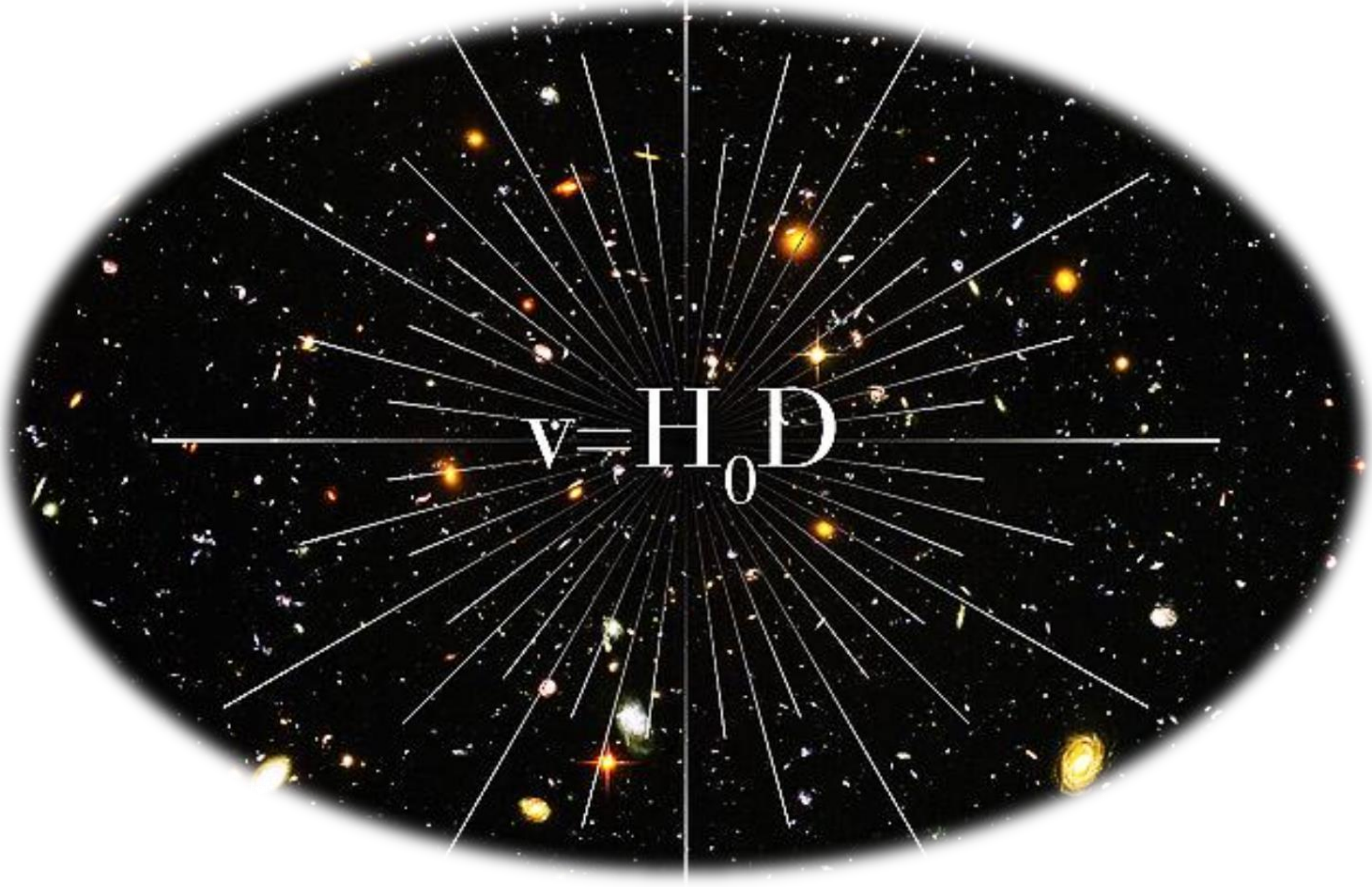

THE EXPANDING UNIVERSE

HKE Society's

A V PATIL ARTS, SCIENCE & COMMERCE COLLEGE
ALAND

A

Project On

Verification of Hubble's Law

(THEORETICAL)

Submitted By

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HKE Society's

A V PATIL ARTS, SCIENCE & COMMERCE COLLEGE ALAND

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CERTIFICATION OF COMPLETION

This is to certify that the project report on "Verification of Hubble's Law"

HKE'S A V PATIL ARTS, SCIENCE & COMMERCE COLLEGE ALAND is carried out under the guidance of Proff. C S Munnolli Lecturer and is submitted to the Department of Physics during the academic year 2021-22

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1. Elementary Ideas

1.1 Universe

From Wikipedia, the free encyclopedia

The **Universe** is commonly defined as the totality of existence,[1][2][3][4] including planets, stars, galaxies, the contents of intergalactic space, and all matter and energy.[5][6] The broadest definition of universe is that it is simply everything, while a narrower definition is that the universe is limited to what can be observed. Similar terms include the *cosmos*, the *world* and *nature*.

Scientific observation of the Universe, the observable part of which is about 93 billion light years in diameter,[7] has led to inferences of its earlier stages. These observations suggest that the Universe has been governed by the same physical laws and constants throughout most of its extent and history. The Big Bang theory is the prevailing cosmological model that describes the early development of the Universe, which in physical cosmology is calculated to have occurred 13.798 ± 0.037 billion years ago.[8][9]

There are various multiverse hypotheses, in which physicists have suggested that the Universe might be one among many universes that likewise exist.[10][11] The farthest distance that it is theoretically possible for humans to see is described as the observable Universe. Observations have shown that the Universe appears to be expanding at an accelerating rate.

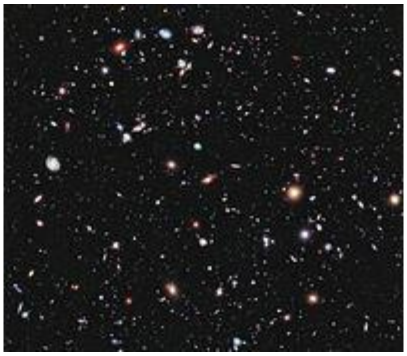
What preceded the gravitational singularity before the Big Bang and the ultimate fate of the Universe remains an unsolved and speculative problem in physics.

1 History

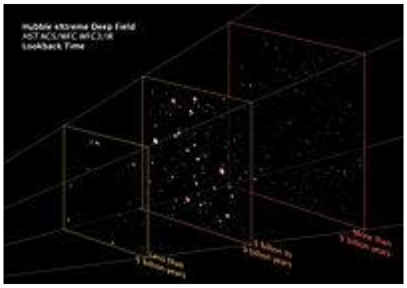
1.1 Observational history

Hubble eXtreme Deep Field (XDF)

XDF size compared to the size of the moon – several thousand galaxies, each consisting of billions of stars, are in this small view.



XDF (2012) view – each light speck is a galaxy – some of these are as old as 13.2 billion years[12] – the Universe is estimated to contain 200 billion galaxies.



XDF image shows fully mature galaxies in the foreground plane – nearly mature galaxies from 5 to 9 billion years ago – protogalaxies, blazing with young stars, beyond 9 billion years.

Throughout recorded history, several cosmologies and cosmogonies have been proposed to account for observations of the Universe. The earliest quantitative geocentric models were developed by the ancient Greek philosophers. Over the centuries, more precise observations and improved theories of gravity led to Copernicus's heliocentric model and the Newtonian model of the Solar System, respectively. Further improvements in astronomy led to the realization that the Solar System is embedded in a galaxy composed of billions of stars, the Milky Way, and that other galaxies exist outside it, as far as astronomical instruments can reach. Careful studies of the distribution of these galaxies and their spectral lines have led to much of modern cosmology. Discovery of the red shift and cosmic microwave background radiation suggested that the Universe is expanding and had a beginning.[13]

1.2 History of the Universe

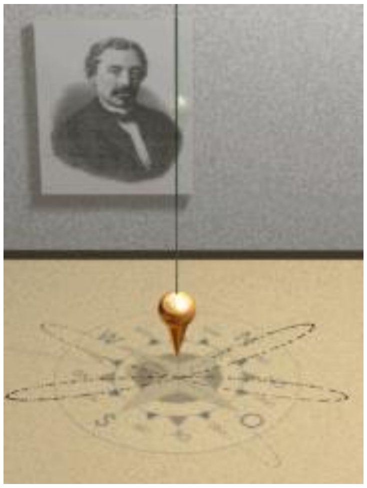
According to the prevailing scientific model of the Universe, known as the Big Bang, the Universe expanded from an extremely hot, dense phase called the Planck epoch, in which all the matter and energy of the observable Universe was concentrated. Since the Planck epoch, the Universe has been expanding to its present form, possibly with a brief period (less than 10^{-32} seconds) of cosmic inflation. Several independent experimental measurements support this theoretical expansion and, more generally, the Big Bang theory. Recent observations indicate that this expansion is accelerating because of dark energy, and that most of the matter in the Universe may be in a form which cannot be detected by present instruments, called dark matter.[14] The common use of the "dark matter" and "dark energy" placeholder names for the unknown entities purported to account for about 95% of the mass-energy density of the Universe demonstrates the present observational and conceptual shortcomings and uncertainties concerning the nature and ultimate fate of the Universe.[15]

On 21 March 2013, the European-led research team behind the Planck cosmology probe released the mission's all-sky map of the cosmic microwave background.[16][17][18][19][20] The map suggests the universe is slightly older than thought. According to the map, subtle fluctuations in temperature were imprinted on the deep sky when the cosmos was about 370,000 years old. The imprint reflects ripples that arose as early, in the existence of the universe, as the first nonillionth of a second. Apparently, these ripples gave rise to the present vast cosmic web of galaxy clusters and dark matter. According to the team, the universe is 13.798 ± 0.037 billion years old,[21][9] and contains 4.9% ordinary matter, 26.8% dark matter and 68.3% dark energy. Also, the Hubble constant was measured to be 67.80 ± 0.77 (km/s)/Mpc.[21][16][17][18][20]

An earlier interpretation of astronomical observations indicated that the age of the Universe was 13.772 ± 0.059 billion years,[22] (whereas the decoupling of light and matter, see CMBR, happened already 380,000 years after the Big Bang), and that the diameter of the observable Universe is at least 93 billion light years or 8.80×10^{26} meters.[23] According to general relativity, space can expand faster than the speed of light, although we can view only a small portion of the Universe due to the limitation imposed by light speed. Since we cannot observe space beyond the limitations of light (or any electromagnetic radiation), it is uncertain whether the size of the Universe is finite or infinite**2**

Etymology, synonyms and definitions

The word *Universe* derives from the Old French word *Univers*, which in turn derives from the Latin word *universum*.[24] The Latin word was used by Cicero and later Latin authors in many of the same senses as the modern English word is used.[25] The Latin word derives from the poetic contraction *Unvorsum* — first used by Lucretius in Book IV (line 262) of his *De rerum natura* (*On the Nature of Things*) — which connects *un, uni* (the combining form of *unus*, or "one") with *vorsum, versum* (a noun made from the perfect passive participle of *vertere*, meaning "something rotated, rolled, changed").[25]



Artistic rendition (highly exaggerated) of a Foucault pendulum showing that the Earth is not stationary, but rotates.

An alternative interpretation of *unvorsum* is "everything rotated as one" or "everything rotated by one". In this sense, it may be considered a translation of an earlier Greek word for the Universe, περιφορά, (*periforá*, "circumambulation"), originally used to describe a course of a meal, the food being carried around the circle of dinner guests.[26] This Greek word refers to celestial spheres, an early Greek model of the Universe. Regarding Plato's Metaphor of the sun, Aristotle suggests that the rotation of the sphere of fixed stars inspired by the prime mover, motivates, in turn, terrestrial change via the Sun. Careful astronomical and physical measurements (such as the Foucault pendulum) are required to prove the Earth rotates on its axis.

A term for "Universe" in ancient Greece was τὸ πᾶν (*tò pán*, The All, Pan (mythology)). Related terms were matter, (τὸ ὅλον, *tò ólon*, see also Hyle, lit. wood) and place (τὸ κενόν, *tò kenón*).[27][28] Other synonyms for the Universe among the ancient Greek philosophers included κόσμος (cosmos) and φύσις (meaning Nature, from which we derive the word physics).[29] The same synonyms are found in Latin authors (*totum*, *mundus*, *natura*)[30] and survive in modern languages, e.g., the German words *Das All*, *Weltall*, and *Natur* for Universe. The same synonyms are found in English, such as everything (as in the theory of everything), the cosmos (as in cosmology), the world (as in the many-worlds hypothesis), and Nature (as in natural laws or natural philosophy).[31]

2.1 Broadest definition: reality and probability

The broadest definition of the Universe is found in *De divisione naturae* by the medieval philosopher and theologian Johannes Scotus Eriugena, who defined it as simply everything: everything that is created and everything that is not created.

2.2 Definition as reality

More customarily, the Universe is defined as everything that exists, (has existed, and will exist)[*citation needed*]. According to our current understanding, the Universe consists of three principles: spacetime, forms of energy, including momentum and matter, and the physical laws that relate them.

2.3 Definition as connected space-time

It is possible to conceive of disconnected space-times, each existing but unable to interact with one another. An easily visualized metaphor is a group of separate soap bubbles, in which observers living on one soap bubble cannot interact with those on other soap bubbles, even in principle. According to one common terminology, each "soap bubble" of space-time is denoted as a universe, whereas our particular space-time is denoted as *the Universe*, just as we call our moon *the Moon*. The entire collection of these separate space-times is denoted as the multiverse.[32] In principle, the other unconnected universes may have different dimensionalities and topologies of space-time, different forms of matter and energy, and different physical laws and physical constants, although such possibilities are purely speculative.

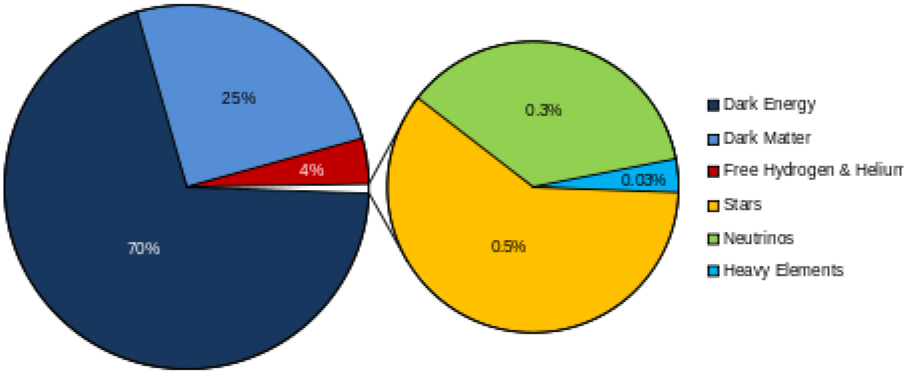
2.4 Definition as observable reality

According to a still-more-restrictive definition, the Universe is everything within our connected space-time that could have a chance to interact with us and vice versa.[*citation needed*] According to the general theory of relativity, some regions of space may never interact with ours even in the lifetime of the Universe, due to the finite speed of light and the ongoing expansion of space. For example, radio messages sent from Earth may never reach some regions of space, even if the Universe would live forever; space may expand faster than light can traverse it.

Distant regions of space are taken to exist and be part of reality as much as we are; yet we can never interact with them. The spatial region within which we can affect and be affected is the observable Universe. Strictly speaking, the observable Universe depends on the location of the observer. By traveling, an observer can come into contact with a greater region of space-time than an observer who remains still, so that the observable Universe for the former is larger than for the latter. Nevertheless, even the most rapid traveler will not be able to interact with all of space. Typically, the observable Universe is taken to mean the Universe observable from our vantage point in the Milky Way Galaxy.

3 Size, age, contents, structure, and laws

The size of the Universe is unknown; it may be infinite. The region visible from Earth (the observable universe) is a sphere with a radius of about 46 billion light years,[33] based on where the expansion of space has taken the most distant objects observed. For comparison, the diameter of a typical galaxy is 30,000 light-years, and the typical distance between two neighboring galaxies is 3 million light-years.[34] As an example, the Milky Way Galaxy is roughly 100,000 light years in diameter,[35] and the nearest sister galaxy to the Milky Way, the Andromeda Galaxy, is located roughly 2.5 million light years away.[36] There are probably more than 100 billion (10^{11}) galaxies in the observable Universe.[37] Typical galaxies range from dwarfs with as few as ten million[38] (10^7) stars up to giants with one trillion[39] (10^{12}) stars, all orbiting the galaxy's center of mass. A 2010 study by astronomers estimated that the observable Universe contains 300 sextillion (3×10^{23}) stars.[40]



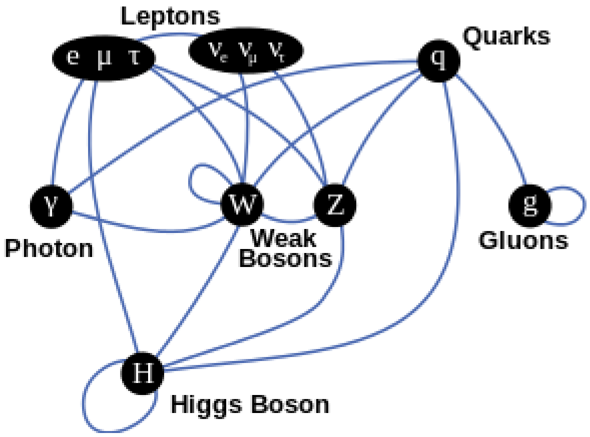
The Universe is believed to be mostly composed of dark energy and dark matter, both of which are poorly understood at present. Less than 5% of the Universe is ordinary matter, a relatively small contribution.

The observable matter is spread homogeneously (*uniformly*) throughout the Universe, when averaged over distances longer than 300 million light-years.[41] However, on smaller length-scales, matter is observed to form "clumps", i.e., to cluster hierarchically; many atoms are condensed into stars, most stars into galaxies, most galaxies into clusters, superclusters and, finally, the largest-scale structures such as the Great Wall of galaxies. The observable matter of the Universe is also spread *isotropically*, meaning that no direction of observation seems different from any other; each region of the sky has roughly the same content.[42] The Universe is also bathed in a highly isotropic microwave radiation that corresponds to a thermal equilibrium blackbody spectrum of roughly 2.725 kelvin.[43] The hypothesis that the large-scale Universe is homogeneous and isotropic is known as the cosmological principle,[44] which is supported by astronomical observations.

The present overall density of the Universe is very low, roughly 9.9×10^{-30} grams per cubic centimetre. This mass-energy appears to consist of 73% dark energy, 23% cold dark matter and 4% ordinary matter. Thus the density of atoms is on the order of a single hydrogen atom for every four cubic meters of volume.[45] The properties of dark energy and dark matter are largely unknown. Dark matter gravitates as ordinary matter, and thus works to slow the expansion of the Universe; by contrast, dark energy accelerates its expansion.

The current estimate of the Universe's age is 13.798 ± 0.037 billion years old.[9] The Universe has not been the same at all times in its history; for example, the relative populations of quasars and galaxies have changed and space itself appears to have expanded. This expansion accounts for how Earth-bound scientists can observe the light from a galaxy 30 billion light years away, even if that light has traveled for only 13 billion years; the very space between them has expanded. This expansion is consistent with the observation that the light from distant galaxies has been redshifted; the photons emitted have been stretched to longer wavelengths and lower frequency during their journey. The rate of this spatial expansion is accelerating, based on studies of Type Ia supernovae and corroborated by other data.

The relative fractions of different chemical elements — particularly the lightest atoms such as hydrogen, deuterium and helium — seem to be identical throughout the Universe and throughout its observable history.[46] The Universe seems to have much more matter than antimatter, an asymmetry possibly related to the observations of CP violation.[47] The Universe appears to have no net electric charge, and therefore gravity appears to be the dominant interaction on cosmological length scales. The Universe also appears to have neither net momentum nor angular momentum. The absence of net charge and momentum would follow from accepted physical laws (Gauss's law and the non-divergence of the stress-energy-momentum pseudotensor, respectively), if the Universe were finite.[48]

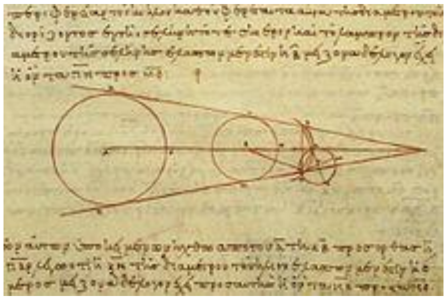


The elementary particles from which the Universe is constructed. Six leptons and six quarks comprise most of the matter; for example, the protons and neutrons of atomic nuclei are composed of quarks, and the ubiquitous electron is a lepton. These particles interact via the gauge bosons shown in the middle row, each corresponding to a particular type of gauge symmetry. The Higgs boson is believed to confer mass on the particles with which it is connected. The graviton, a supposed gauge boson for gravity, is not shown.

The Universe appears to have a smooth space-time continuum consisting of three spatial dimensions and one temporal (time) dimension. On the average, space is observed to be very nearly flat (close to zero curvature), meaning that Euclidean geometry is experimentally true with high accuracy throughout most of the Universe.[49] Spacetime also appears to have a simply connected topology, at least on the length-scale of the observable Universe. However, present observations cannot exclude the possibilities that the Universe has more dimensions and that its spacetime may have a multiply connected global topology, in analogy with the cylindrical or toroidal topologies of two-dimensional spaces.[50]

The Universe appears to behave in a manner that regularly follows a set of physical laws and physical constants.[51] According to the prevailing Standard Model of physics, all matter is composed of three generations of leptons and quarks, both of which are fermions. These elementary particles interact via at most three fundamental interactions: the electroweak interaction which includes electromagnetism and the weak nuclear force; the strong nuclear force described by quantum chromodynamics; and gravity, which is best described at present by general relativity. The first two interactions can be described by renormalized quantum field theory, and are mediated by gauge bosons that correspond to a particular type of gauge symmetry. A renormalized quantum field theory of general relativity has not yet been achieved, although various forms of string theory seem promising. The theory of special relativity is believed to hold throughout the Universe, provided that the spatial and temporal length scales are sufficiently short; otherwise, the more general theory of general relativity must be applied. There is no explanation for the particular values that physical constants appear to have throughout our Universe, such as Planck's constant *h* or the gravitational constant *G*. Several conservation laws have been identified, such as the conservation of charge, momentum, angular momentum and energy; in many cases, these conservation laws can be related to symmetries or mathematical identities.

Astronomical models



Aristarchus's 3rd century BCE calculations on the relative sizes of from left the Sun, Earth and Moon, from a 10th century AD Greek copy

Astronomical models of the Universe were proposed soon after astronomy began with the Babylonian astronomers, who viewed the Universe as a flat disk floating in the ocean, and this forms the premise for early Greek maps like those of Anaximander and Hecataeus of Miletus.

Later Greek philosophers, observing the motions of the heavenly bodies, were concerned with developing models of the Universe based more profoundly on empirical evidence. The first coherent model was proposed by Eudoxus of Cnidos. According to Aristotle's physical interpretation of the model, celestial spheres eternally rotate with uniform motion around a stationary Earth. Normal matter, is entirely contained within the terrestrial sphere. This model was also refined by Callippus and after concentric spheres were abandoned, it was brought into nearly perfect agreement with astronomical observations by Ptolemy. The success of such a model is largely due to the mathematical fact that any function (such as the position of a planet) can be decomposed into a set of circular functions (the Fourier modes). Other Greek scientists, such as the Pythagorean philosopher Philolaus postulated that at the center of the Universe was a "central fire" around which the Earth, Sun, Moon and Planets revolved in uniform circular motion.[60] The Greek astronomer Aristarchus of Samos was the first known individual to propose a heliocentric model of the Universe. Though the original text has been lost, a reference in Archimedes' book The Sand Reckoner describes Aristarchus' heliocentric theory. Archimedes wrote: (translated into English)

You King Gelon are aware the 'Universe' is the name given by most astronomers to the sphere the center of which is the center of the Earth, while its radius is equal to the straight line between the center of the Sun and the center of the Earth. This is the common account as you have heard from astronomers. But Aristarchus has brought out a book consisting of certain hypotheses, wherein it appears, as a consequence of the assumptions made, that the Universe is many times greater than the 'Universe' just mentioned. His hypotheses are that the fixed stars and the Sun remain unmoved, that the Earth revolves about the Sun on the circumference of a circle, the Sun lying in the middle of the orbit, and that the sphere of fixed stars, situated about the same center as the Sun, is so great that the circle in which he supposes the Earth to revolve bears such a proportion to the distance of the fixed stars as the center of the sphere bears to its surface.

Aristarchus thus believed the stars to be very far away, and saw this as the reason why there was no visible parallax, that is, an observed movement of the stars relative to each other as the Earth moved around the Sun. The stars are in fact much farther away than the distance that was generally assumed in ancient times, which is why stellar parallax is only detectable with telescopes. The geocentric model, consistent with planetary parallax, was assumed to be an explanation for the unobservability of the parallel phenomenon, stellar parallax. The rejection of the heliocentric view was apparently quite strong, as the following passage from Plutarch suggests (On the Apparent Face in the Orb of the Moon):

THE HUBBLE LAW

Objective

To derive a value for the Hubble constant and the age of the universe.

Introduction and Overview

In the 1920's, Edwin P. Hubble discovered that distant galaxies were all moving away from the Milky Way (and th Local Group). Not only that, the farther away he observed, the faster the galaxies were receding. He found the relationship that is now known as Hubble's Law: the recessional velocity of a galaxy is proportional to its distance from us. The equation looks like this:

$$\mathbf{v = H_o * d,}$$

where **v** is the galaxy's velocity (in km/sec), **d** is the distance to the galaxy (in *mega parsecs*; 1 Mpc = 1 million parsecs), and **H_o** is the proportionality constant, called "The Hubble constant." This equation is telling us that a galaxy moving away from us twice as fast as another galaxy will be twice as far away, a galaxy moving away from us three times as fast will be three times farther away, and so on.

The value for the Hubble constant, which gives the age of the Universe, has been an area of ongoing debate. Even the most recent observations using the Hubble space telescope have not silenced the feuding sides. Before the HST observations, one group insisted the Hubble constant was about 100 km/sec/Mpc (giving an age for the Universe of around 10 billion years) while the other group claimed a value of 50 km/sec/Mpc (20 billion years). Although the sides moved a bit closer with additional observations, 80 km/sec/Mpc versus 60 km/sec/Mpc (12 billion years versus 17 billion years), both groups insisted that their value for the Hubble constant was, in fact, the correct value. The question for the value of the Hubble constant and the age of the Universe may, however, be resolved. Recent results (February 2003) from the Wilkinson Microwave Anisotropy Probe (WMAP) indicate that the Universe is 13.7 billion years old (Hubble constant = 73 km/sec/Mpc), with just about a 1% margin of error. Astounding results indeed!

Why such a heated debate over a single number? The Hubble Constant is one of the most important numbers in cosmology because it is a measure of the age of the universe. This long-sought-after number indicates the rate at which the universe is expanding, the velocity stemming from the primordial "Big Bang." The Hubble Constant can be used to determine the intrinsic brightness and masses of stars in nearby galaxies, examine those same properties in more distant galaxies and galaxy clusters, deduce the amount of dark matter present in the Universe, obtain the scale size of faraway galaxy clusters, and serve as a test for theoretical cosmological models.

In the short time we have remaining in this quarter, we will enter this debate as we work to determine **our** value for the Hubble constant and get from it the age of the Universe. Read through the following summary of the steps to be taken and get an overview of what is involved. You won't need to stay up all night making the observations, but you will need to decide which galaxies to use. Once your galaxies are chosen, you will move to finding the recessional velocity for each galaxy and its distance. Your data analysis will lead to your value for the Hubble constant, the uncertainty in the value, and the age of the Universe. This lab uses much of the knowledge you have gained over the past few weeks. Ready? Let's begin.

The Steps towards the Hubble Constant and the Age of the Universe

Step 1: Getting to Know the Galaxies

Our first step will be to become familiar with the images and the spectra of the galaxies with which we will be working. These images and spectra are **real** data, and were obtained using a CCD (charge-coupled device) on a couple of large (2 - 4 meter), ground-based telescopes. You will be sketching, classifying, and describing each galaxy.

Step 2: Selecting Your Galaxies

Out of the 27 images and spectra of galaxies that are available for analysis, you will need to choose 15 to analyze (including those that may have been preselected). We want to use galaxies that have similar looks and characteristics so that we can be relatively sure we are using galaxies that are all the same actual size. We do this by seeing how they **look** and what their **spectra** are like. We want spiral galaxies; we do not want elliptical galaxies. The reason why is explained below.

Step 3: Finding the velocity of each galaxy

The velocity is relatively easy for us to measure using the Doppler effect. An object in motion (in this case, being carried along by the expansion of space itself) will have its radiation (light) shifted in wavelength. For velocities much smaller than the speed of light, we can use the regular Doppler formula:

$$\frac{(\lambda-\lambda o)}{\lambda o} = \frac{v}{c}$$

λ : Measured wavelength
λo: Original (rest) wavelength
v : Speed of object
c : Speed of light

The quantity on the left side of this equation is usually called the **redshift**, and is denoted by the letter **z**. The **velocity** of the galaxy is determined by measuring the redshift of spectral lines in the **spectrum** of the galaxy. The full optical spectrum of the galaxy is shown at the top of the web page containing the spectrum of the galaxy being measured. Below it are enlarged portions of the same spectrum, in the vicinity of some common galaxy spectral features: the "K and H" lines of ionized calcium and the *H-alpha* line of hydrogen.

Step 4: Finding the distance to each galaxy

The next step is to determine the distances to galaxies. For nearby galaxies, we can use standard candles such as Cepheid variables or white-dwarf supernovae. But, for very distant galaxies, we must rely on more indirect methods. **The key assumption for this lab is that galaxies of similar Hubble type have similar actual sizes**. This is known as "the standard ruler" assumption. We must first calibrate the actual size by using a galaxy to which we know the true distance. We are looking for galaxies in the sample that are spiral galaxies, as we would use nearby spiral galaxies (M31 the Andromeda galaxy, for example) to calibrate the distances. We **know** the distances to many nearby galaxies through observations of the Cepheid variables in them. Then, to determine the distance to very distant, but similar galaxies, one would only need to measure their **apparent** (angular) sizes, and use the small-angle formula.

Step 5: Data Analysis

Here is the step where you determine the Hubble constant and the uncertainty in that constant. You will be graphing the distance to each galaxy in megaparsecs (*x-axis*) versus the recessional velocity of that galaxy in kilometers per second (*y-axis*) and calculating the slope of the data -- your Hubble constant. The uncertainty in your constant is the uncertainty in the slope: what is the steepest your slope could be (highest value for the Hubble constant) and what is the shallowest (lowest value)?

With your value for the Hubble constant in hand, you are ready to calculate the age of the Universe using both a simple model for the expansion and a more realistic model that includes gravity.

Procedure

Step 1: Getting to Know the Galaxies

Our first step will be to become familiar with the images and the spectra of the galaxies with which we will be working. These images and spectra are **real** data, and were obtained using a CCD (charge-coupled device) on a couple of large (2 - 4 meter), ground-based telescopes.

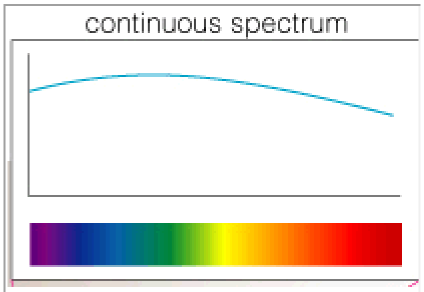
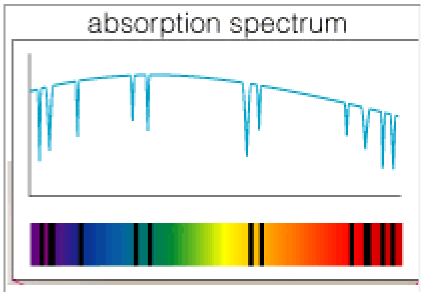
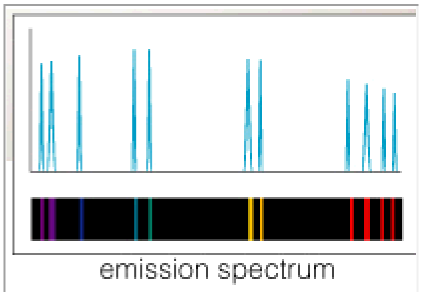
The Images

Examine closely the 27 galaxies linked from the page showing all of the galaxies. Note any substructure, irregularities, or other defining characteristics for each galaxy. These features may be difficult to see for the more distant galaxies. On the galaxy overview answer sheet (PDF), **sketch each galaxy**, and give your best guess of its general Hubble type (spiral, barred spiral, elliptical, irregular).

The Spectra

Examine closely the 27 spectra shown on these full spectra pages that also include images of the galaxies. You are looking at the **relative intensity** of the total light radiated from each galaxy as a function of wavelength. The overall shape or curve of each spectrum is due to the continuous spectra from those stars that dominate the light coming from the galaxy (thermal radiation). Where you see dips in the spectrum of a galaxy, that is where radiation is being absorbed. Where you see sharp spikes in the spectrum of a galaxy, that is where radiation is being emitted. Unlike our "idealized" spectra of earlier in the quarter where we examined individual stars, the spectra from these galaxies reflect the total of all of the light from all of the objects in them.

Figure 1: A Short Review of the Kinds of Spectra

		
Continuous spectrum of thermal radiation	Absorption spectrum showing absorption of light from thermal radiation passing	Emission spectrum of hot, thin gas seen against a cooler

	through a cooler, thin gas.	background.

There are a couple of features you should especially note when trying to decipher these spectra:

1. Not all of the "jiggly" lines come from the light of the galaxy. Each spectrum contains noise; we just cannot get away from it. You should notice that some of the spectra are much "noisier" than other spectra. This noise tends to hamper accurate identification of some of the lines.
2. Most of the spectra show strong hydrogen emission lines along with some absorption lines. Note that the "relative intensity" axes are not all at the same scale. Some spectra will look "flat", when, in fact, the scaling had to be adjusted to accommodate an intense, hydrogen emission line, usually the one at 656.28 nm (6562.8 Angstroms). The relative intensity for some spectra ranges from 0 to 1.2; for others, from 0 to 15.
3. Some spectra show **only** absorption lines, or absorption lines with very weak hydrogen emission lines.
4. What you should be looking for are absorption lines of ionized calcium, lines designated by "H" and "K" [rest wavelengths of 396.85 and 393.37 nm (3968.5 and 3933.7 Angstroms)] and the emission of the *H-alpha* line of hydrogen [rest wavelength of 656.28 nm (6562.8 Angstroms)]. **Remember**: these spectra are of galaxies that are moving away from us and so the lines are going to be **redshifted** (shifted towards longer wavelengths); some, you will find out, by a large amount.

After looking closely at the corresponding spectrum for each galaxy, write a short description of the spectrum in the space provided on the galaxy overview sheet. You will be using these sketches, classifications and descriptions shortly to eliminate some of the galaxies from further consideration.

What these spectra tell us

These plots of "jiggly lines" are telling us all about these galaxies, just as stellar spectra tell us all about stars. Remember the primary objects found in spiral galaxies: stars of all ages, masses, and composition; dust; and HII regions. We expect, because the bright HII regions and massive OB stars will dominate the light of a spiral galaxy, to see strong emission lines of hydrogen.

On the other hand, most elliptical galaxies contain old, cool stars. There is little or no free dust and gas in ellipticals, and certainly no massive star formation. We expect to see absorption lines dominating the spectra of elliptical galaxies, especially lines of ionized calcium (CaII H & K) and hydrogen. The spectrum of a galaxy will represent the total light coming from those objects that are **contributing the most** to the light of the galaxy. These objects will be those that far outnumber other objects, or are the most luminous (red giants), or both.

Step 2: Selecting Your Galaxies

The critical assumption

We want to **work only with spiral galaxies** (barred spirals will also be okay), and not elliptical galaxies. Why? Recall that if we see a galaxy that is 1/2 or 1/3 the angular (apparent) size of another galaxy, we would like to be able to state that that galaxy is 2 times or 3 times farther away. To do this, we must assume that galaxies of similar Hubble type are similar in actual size.

A secondary criterion is similar spectral characteristics. Although spiral galaxies may or may not have hydrogen lines in emission (especially H-alpha), we expect that elliptical galaxies will show no emission at all. As you review your classifications of the galaxies and your descriptions of the spectra, do you see any pattern or correlation? You should use this pattern or correlation in your decision to "keep or toss."

Selecting the Galaxies

In the last column of the galaxy and spectra overview table, mark down your decision to keep or toss that particular galaxy. You should plan to keep 15 galaxies (including any pre-selected ones) to give you enough galaxies to work with in deriving the Hubble constant. **Once you eliminate a galaxy, you do not need to do anything more with that galaxy.**

Note: to make this task a bit faster, 5 galaxies have already been selected, and a few already eliminated. You will need to choose about 10 more galaxies and eliminate the rest.

After your selection process is complete, answer this question for yourself: "Based upon these images, what do I foresee as possible problems in measuring the angular diameters of the galaxies?"

Step 3: Finding the velocity of each galaxy

The velocity is relatively easy for us to measure using the Doppler Effect. An object in motion (in this case, being carried along by the expansion of space itself) will have its radiation (light) shifted in wavelength. For velocities much smaller than the speed of light, we can use the regular Doppler formula:

$$\frac{(\lambda - \lambda o)}{\lambda o} = \frac{v}{c}$$

λ : Measured wavelength
λo: Original (rest) wavelength
v : Speed of object
c : Speed of light

The quantity on the left side of this equation is usually called the **redshift**, and is denoted by the letter **z**.

The formula for redshift should remind you of the process where you calculated your percentage error: [(your value) - (true value)] / (true value). Thus, we can view the redshift (at least for those galaxies with a recessional velocity much less than the speed of light) as a "percentage" wavelength shift. It is a measure of the ratio between the velocity of the galaxy and the speed of light.

For this lab, all wavelengths will be measured in Ångstroms (Å), and we will approximate the speed of light at 300,000 km/sec. Thus, we can determine the velocity of a galaxy from its spectrum: we simply measure the (shifted) wavelength of a known absorption line and solve the equation **v = z * c**.

For Example: *A certain absorption line that is found at 5000Å in the lab (rest wavelength) is found at 5050Å when analyzing the spectrum of a particular galaxy. We first calculate z:*

redshift = [(measured wavelength) - (rest wavelength)] / (rest wavelength)

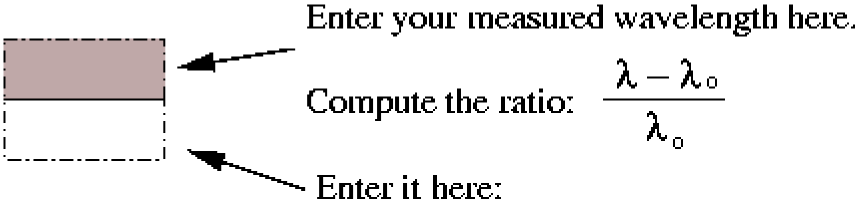
$$\mathbf{z} = \frac{\lambda - \lambda_o}{\lambda_o}$$

*We find that z = 50/5000 = 0.001 and conclude that this galaxy is moving with a velocity v = 0.001 * c = 3000 km/sec away from us.*

Measuring the Spectral Lines

- The **velocity** of the galaxy is determined by measuring the redshift of spectral lines in the **spectrum** of the galaxy. The full optical (visual wavelengths) spectrum of the galaxy is shown at the top of the web page containing the spectrum of the galaxy being measured (see link below). Below it are enlarged portions of the same spectrum, in the vicinity of some common galaxy spectral features: the hydrogen transitions hydrogen-alpha (656.28 nm, 6562.8 Å), hydrogen-beta (486.13 nm, 4861.3 Å), hydrogen-delta (410.17 nm, 4101.7 Å) as well as the "K and H" lines of ionized calcium (393.37 and 396.85 nm, 3933.7 and 3968.5 Å). The enlarged portions of the same spectrum are "**clickable**" and will return a wavelength value corresponding to where you "clicked." Take a brief look at the spectrum for NGC1357 and the analysis of the spectrum. You should try to use similar logic when measuring the rest of your selected galaxies.

- You are now ready to do your measurements.
    1. Make sure you have a copy of the Data Table from the set of student answer sheets. (PDF)
    2. Note that for each galaxy there are two lines of data under each "spectral lines" column. The first line contains the measured wavelength. The second line contains the calculated redshift.



    3. Start with NGC 1357 to see if you can duplicate or come close to the values discussed under the analysis. Note how the data table has been filled in for NGC 1357, and make sure you understand what data goes where and what calculations are being done.
    4. Move on to the next galaxy, NGC 1832. Note that this galaxy, too, has been measured for you. Again, see if your measurements mimic these data.
    5. Use the velocities of these two galaxies as part of the 15 velocities needed to calculate the Hubble constant (leaving you only 13 to do!).
    6. Now move on to the next galaxy that you have selected. Starting with the calcium lines, measure the wavelength of the **same but shifted** line by clicking at the middle of the spectral line (i.e. at the "greatest depth" of absorption or "peak" of emission)in the spectrum of the galaxy. Write this wavelength in the box below the appropriate line designation in your data table. (Note: It is due to the peculiarities of each galaxy that some spectral lines are absent, or show up in **emission** instead of **absorption**.)
    7. Do this for the rest of your selected galaxies, trying to measure the shifts of at least 3 lines in the spectrum. For each of your galaxies, you will measure, calculate redshifts, average redshifts, derive a velocity (remember: $v = z * c$). These are the "**y**" values for your graph. Then you will be ready to find the "**x**" values -- the corresponding distances.

8. For the galaxies **not used** simply cross out the row next to the galaxy number.

Here is the listing of the files that contain the <u>real data for the 27 galaxies</u>.

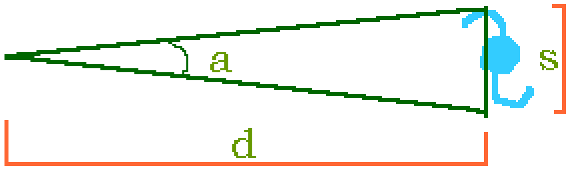
Step 4: Finding the distance to each galaxy

A trickier task is to determine the distances to galaxies. For nearby galaxies, we can use standard candles such as Cepheid variables or Type I supernovae. But, for very distant galaxies, we must rely on more indirect methods. **The key assumption for this lab is that galaxies of similar Hubble type are, in fact, of similar actual size, no matter how far away they are.** This is known as "the standard ruler" assumption. We must first calibrate the actual size by using a galaxy to which we know the true distance. We are looking for galaxies in the sample that are Sb galaxies, as we would use the nearby Sb galaxy, M31 the Andromeda galaxy, to calibrate the distances. We **know** the distance to the Andromeda galaxy through observations of the Cepheid variables in it. Then, to determine the distance to more distant, similar galaxies, one would only need to measure their **apparent** (angular) sizes, and use the following approximation for small angles:

$$a = s / d$$

or:

$$d = s / a$$

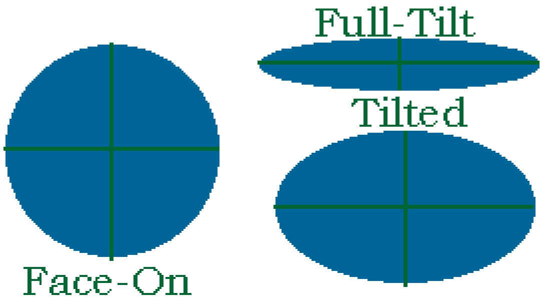


where**a** is the measured angular size (in radians), **s** is the galaxy's true size (diameter), and **d** is the distance to the galaxy.

Measuring the Galaxies

- **It is up to you to decide the criteria you will use in measuring these galaxies. It is suggested that you try to measure as far out as you can see any fuzzy disk.**
- The **angular size** of the galaxy is measured by using its **image**. Note that the images used in this lab are **negatives**, so that bright objects -- such as stars and galaxies -- appear dark. Note also that there may be more than one galaxy in the image; the galaxy of interest is always the one closest to the center.
- To measure the size, simply move the mouse and click on opposite ends of the galaxy, along its **longest** part. (You will need to make a total of **two** clicks.)

Take a look at this schematic of a galaxy viewed from three different angles. Thought question: We assume that the spirals are all round, and that their different shapes are simply because we are viewing them from different angles. When measuring the angular sizes of the galaxies, why should you measure along the ***longest*** *axis only?*



The angular size of the galaxy (in *milliradians*; 1 mrad = 0.057 degrees = 206 arcseconds) will be displayed; write this number down on your data table given in the student answer sheets, under "Galaxy Size."

If, at any point, you make an error while you're measuring (e.g. a mis-click), simply click on the "back" button of your web browser and take the measurement again.

Here is the listing of the files containing the <u>real data for the 27 galaxies</u>.

Checking Your Data

It would be a good idea to have your instructor looks at your data now, before you do a ton of calculations. You wouldn't want to spend hours of your time only to discover that you made mistakes in steps 3 and 4.

Initial Calculations

If you feel confident of your data, then you are ready for the preliminary calculations:

<u>Velocity Determination</u>

For **each measured line** calculate the (<u>redshift **z**</u>), and enter this value in the box under the measured wavelength. Then take the average redshift of the measured lines for each galaxy, and enter it on the appropriate column. Finally, use this average redshift to calculate the velocity of the galaxy using the modified Doppler-shift formula:

$$\mathbf{v = c * z}$$

<u>Distance Determination</u>

Determine the distance (in Mpc) to each galaxy using the following, revised version of the small angle formula. Recall, we have had to make an important assumption: all of these galaxies are about the same actual size. Once you have the angular diameter in *mrad* (and with some adjusting of units), just take the **actual size** of each galaxy -- 22 kpc -- and divide it by the measured angular diameter. For example, if one of the galaxies had a measured angular diameter of 0.50 *mrad*, 22 / 0.50 = 44 Mpc.

Details for the manipulation of the units to come out with the correct distances

From calibrations, we know that galaxies of the type used in this lab are about **22 kpc** (1 kiloparsec = 1000 pc) across. We may then find the distance to the galaxies:

distance (kpc) = size (kpc) / a (rad)

or equivalently, upon multiplying the left side by 1000 and dividing the right side by 0.001 (which is exactly the same thing):

distance (Mpc) = size (kpc) / a (mrad)

Note that we now have the equation in a form where we can simply substitute the size in kpc (22) and divide it by the angle returned by our measurements (already in *mrad*).

THE HUBBLE LAW

Data Analysis and Questions

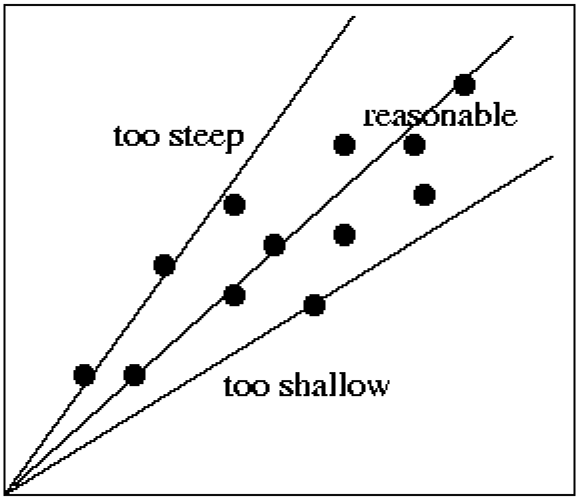
Step 5: Data Analysis

Determining the Hubble constant

1. Using the graph provided in the the student answer sheets), graph your data with **distance in megaparsecs (Mpc) on the x-axis**, and **velocity in kilometers per second (km/s) on the y-axis**. Draw a straight line that best fits the points on the graph; remember that this line **must pass through the origin** (the 0,0 point). Measure the slope of this line (rise/run), this is your value of the Hubble constant, in the units of *km/sec/Mpc*. Please show all calculations and record the slope (the Hubble constant) in the Table of Results (under Step 6).

Determining the uncertainty in the Hubble constant

2. Hubble's Law predicts that galaxies should lie on a straight line when plotted on a graph of distance vs. velocity. Your data probably do not make a perfectly straight line, and you most likely had to make a guess as to where to draw your line. One simple



way to estimate the uncertainty in the value of H_0 is to draw the steepest *reasonable* line *and* the shallowest *reasonable* line on the graph, and calculate their slopes. **Half of the difference** between these two slopes would be your uncertainty. Record this number in the table.

Please show all calculations and results on your <u>Table of Results</u> (PDF) given with the answer sheets.

Determining the Age of the Universe:

3. Maximum age of the Universe

If the universe has been expanding since its beginning at a constant speed, the universe's age would simply be $1/H_0$.

a. Find the inverse of your value of H_0.
b. Multiply the inverse by 3.09×10^{19} km/Mpc to cancel the distance units.
c. Since you now have the age of the Universe in seconds, divide this number by the number of seconds in a year: 3.16×10^{7} sec/yr.

EXAMPLE:
Your Hubble constant is 75 km/sec/Mpc,
then 1/75 = 0.0133 = 1.33×10^{-2}

$(1.33 \times 10^{-2}) \times (3.09 \times 10^{19}) = 4.12 \times 10^{17}$

(4.12×10^{17}) divided by $(3.16 \times 10^{7}) = 1.3 \times 10^{10}$

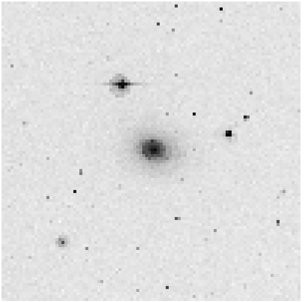
This is 1.3×10^{10} years,
or 13×10^{9} years,
or 13 billion years.

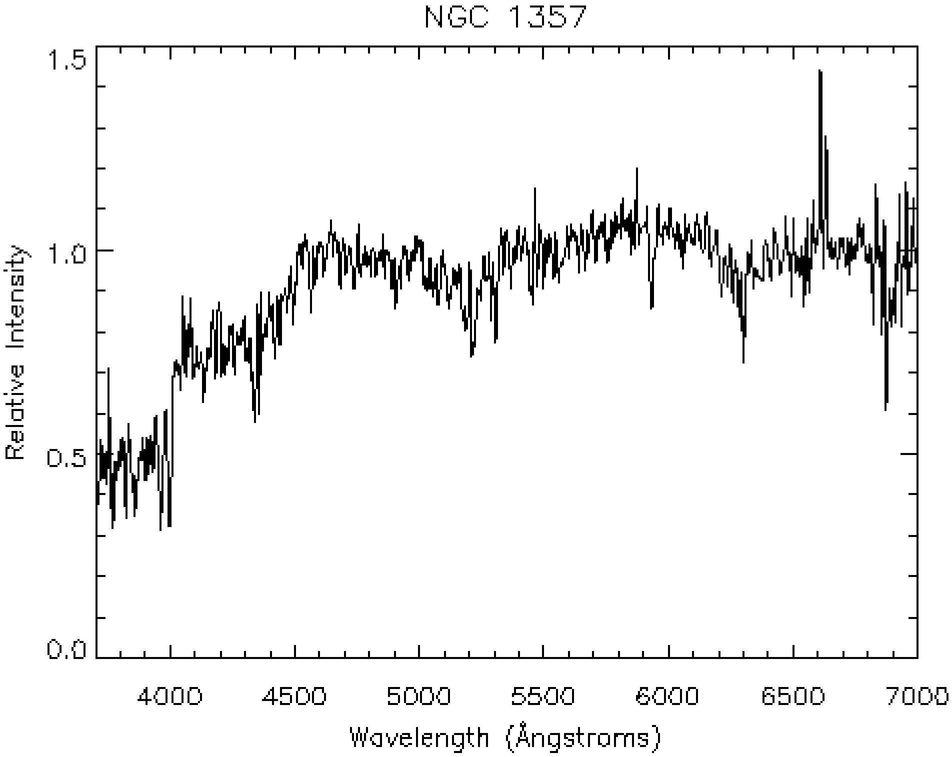
4. This age represents a very simple model for the expansion of the universe, and is the **maximum** age the universe can be.

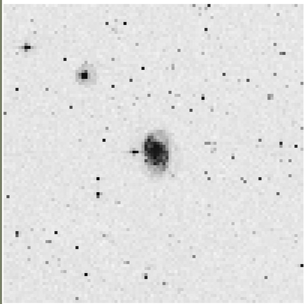
5. The age of the Universe with gravity

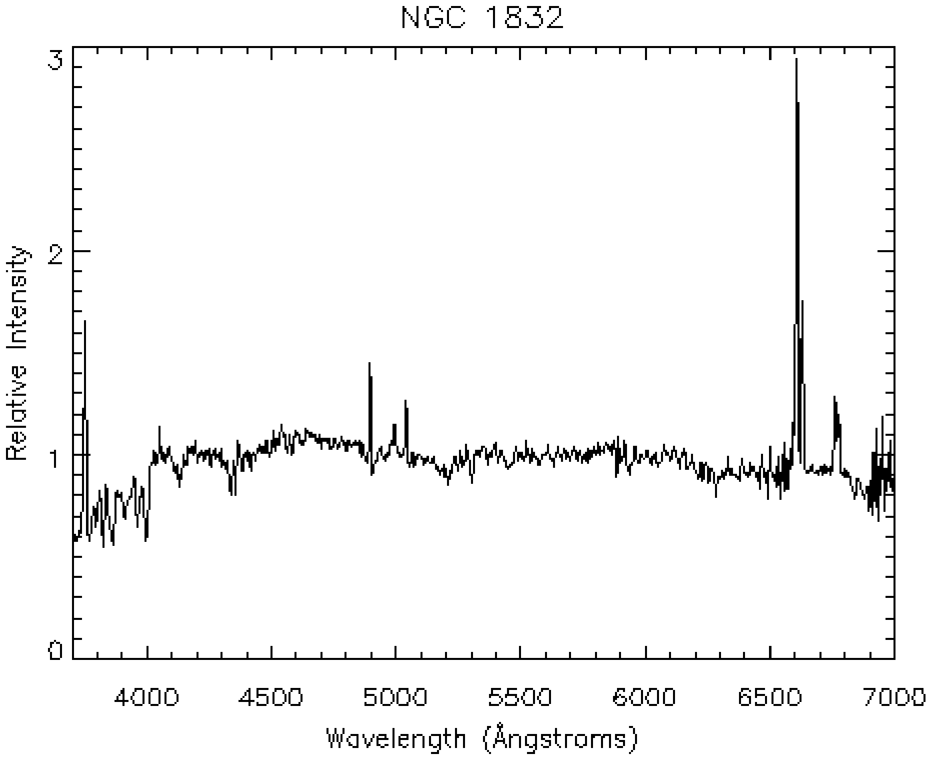
A better model would account for the deceleration caused by gravity. Models like this predict the age of the universe to be: **$t = (2/3)*(1/H_0)$**, or 2/3 of the maximum age of the Universe. Re-calculate the age using this relation (**don't forget to take care of the conversion of units!**), and record in the table. Remember to show all calculations.

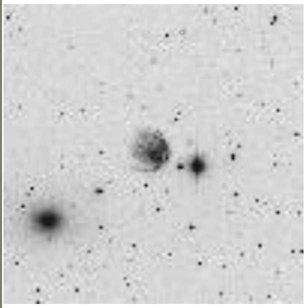
Images and Spectra of the Galaxies

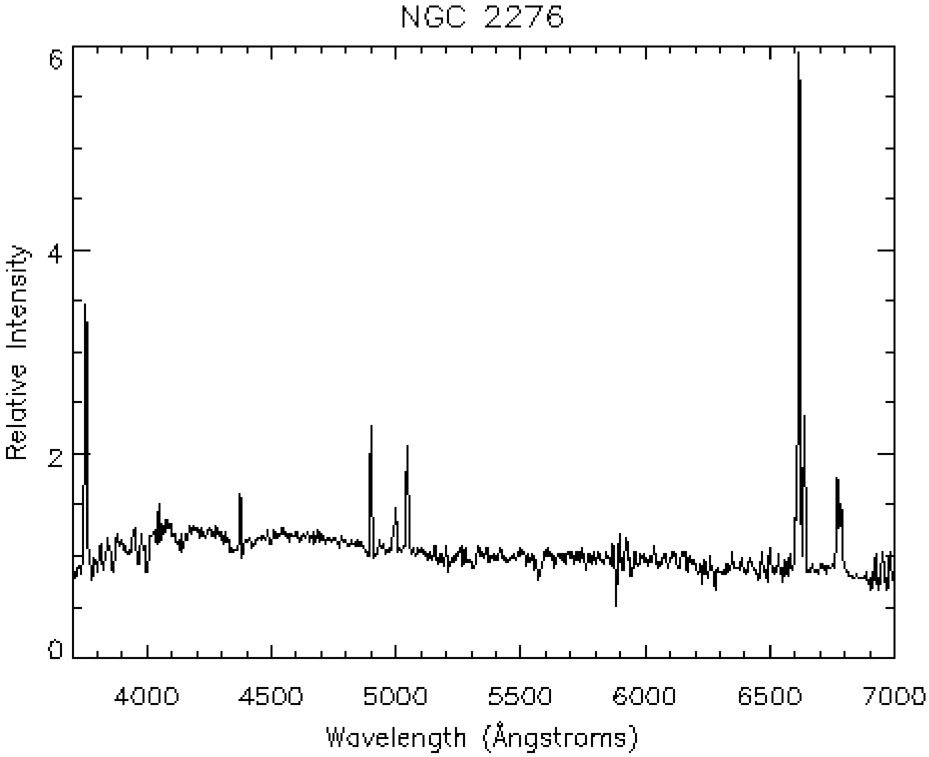


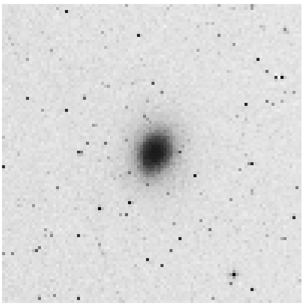


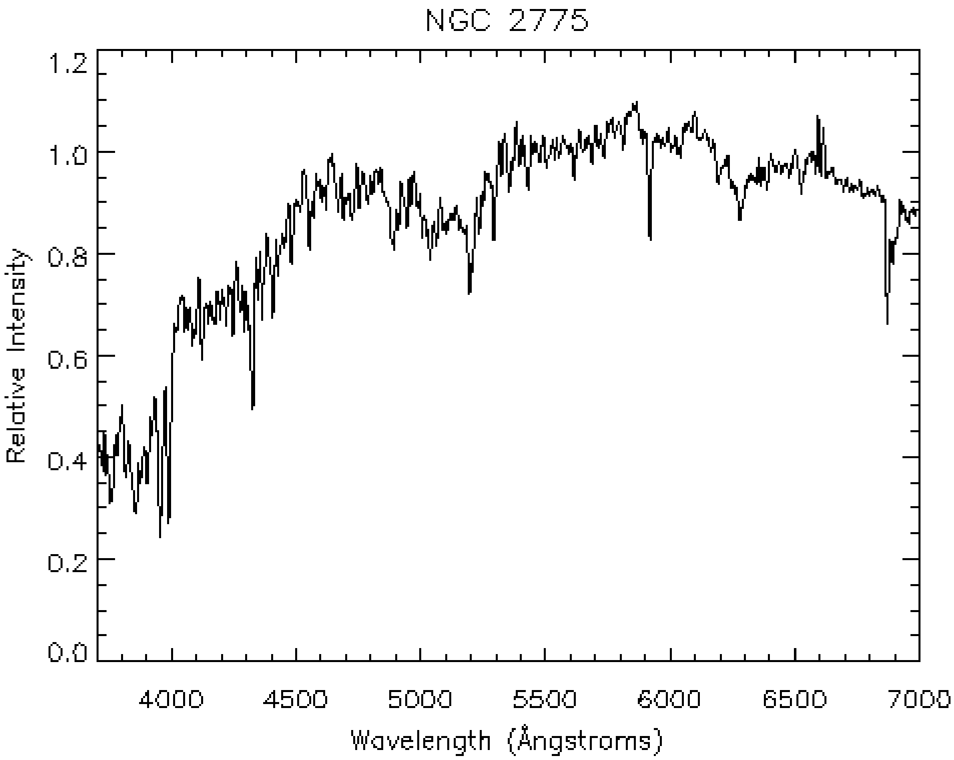


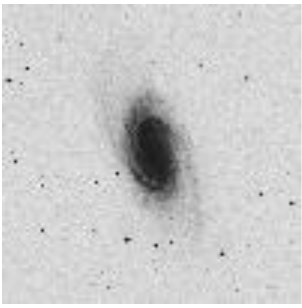

NGC 1832

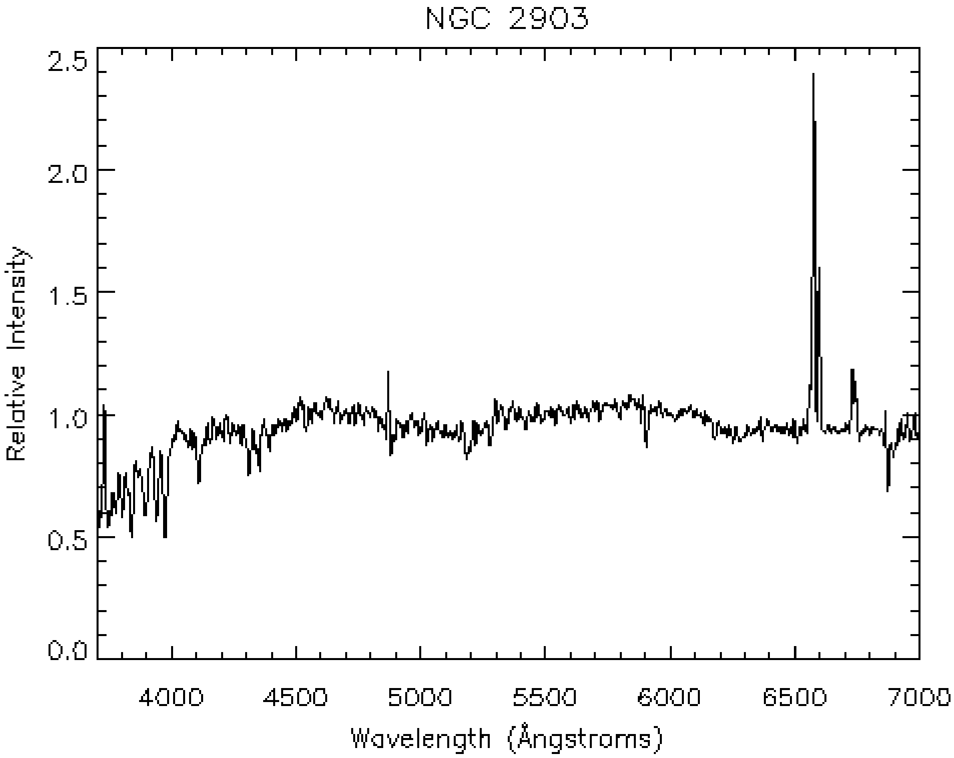



NGC 2276




NGC 2775
Relative Intensity
0.0
0.2
0.4
0.6
0.8
1.0
1.2
4000
4500
5000
5500
6000
6500
7000
Wavelength (Ångstroms)




NGC 2903
Relative Intensity
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0.5
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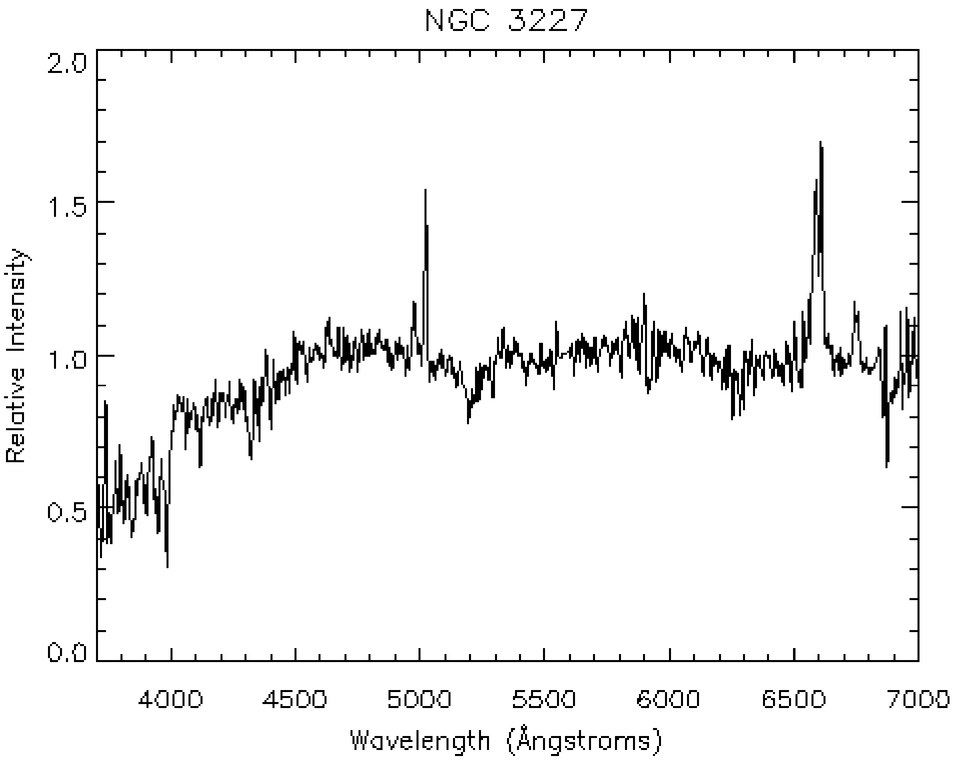
1.5
2.0
2.5
4000
4500
5000
5500
6000
6500
7000
Wavelength (Ångstroms)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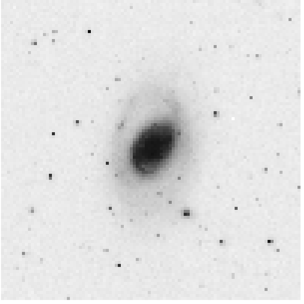
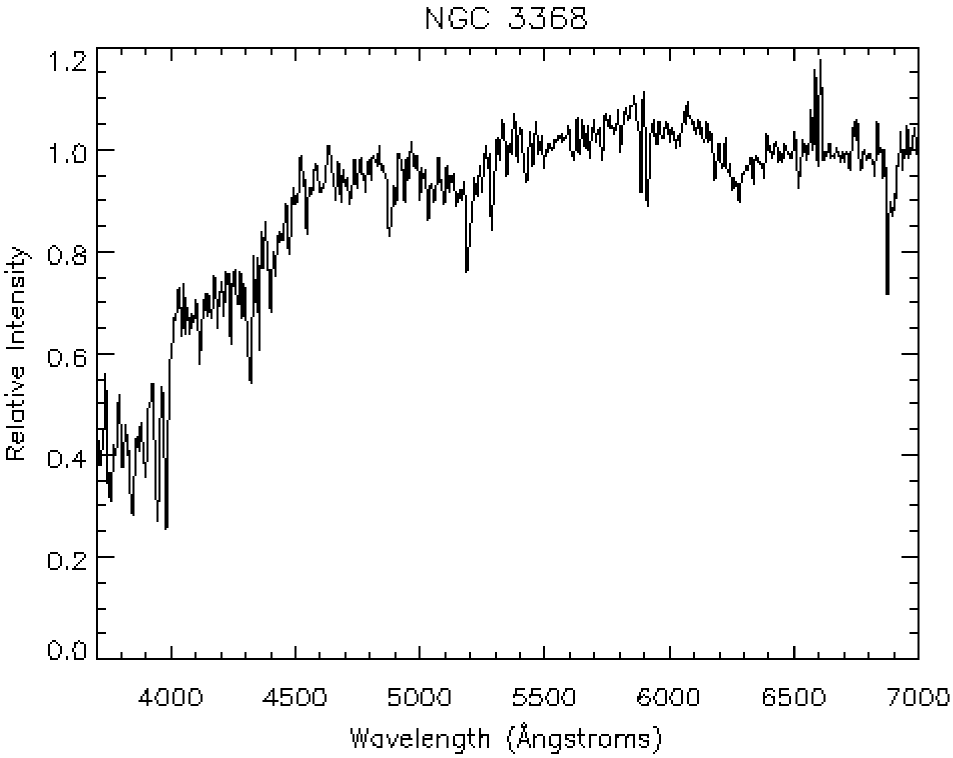
5000
5500
6000
6500
7000
Wavelength (Ångstroms)

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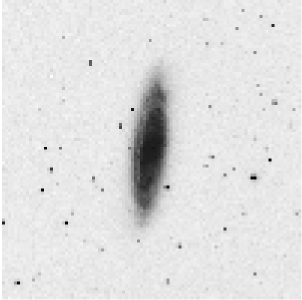
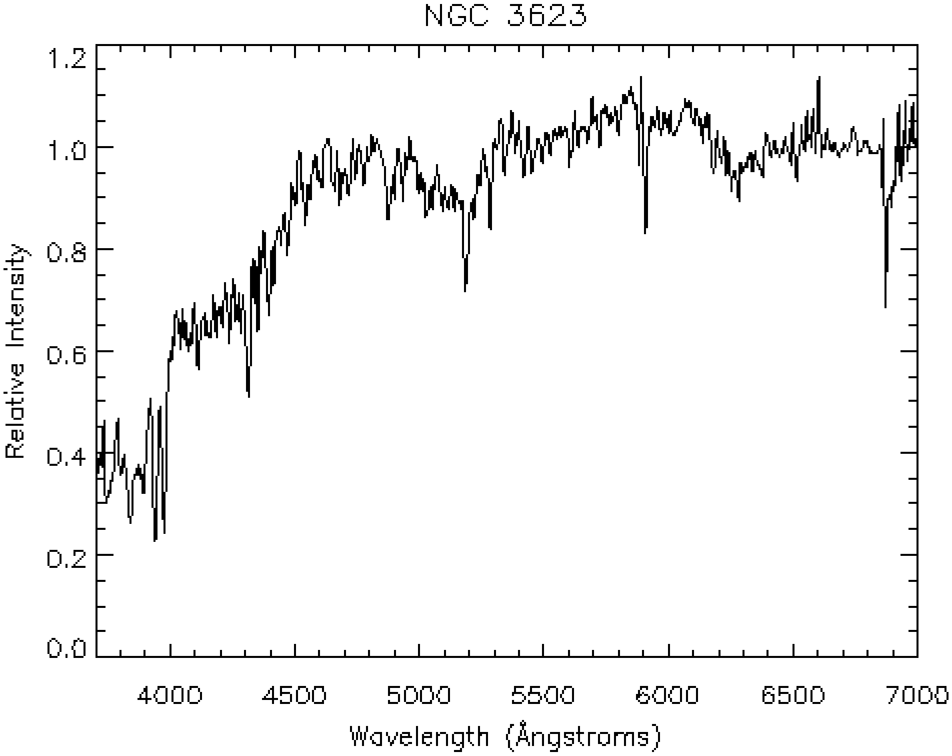
4500
5000
5500
6000
6500
7000
Wavelength (Ångstroms)




NGC 3623
Relative Intensity
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0.8
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1.2
Wavelength (Ångstroms)
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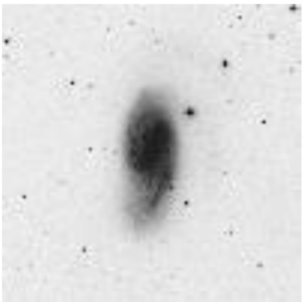
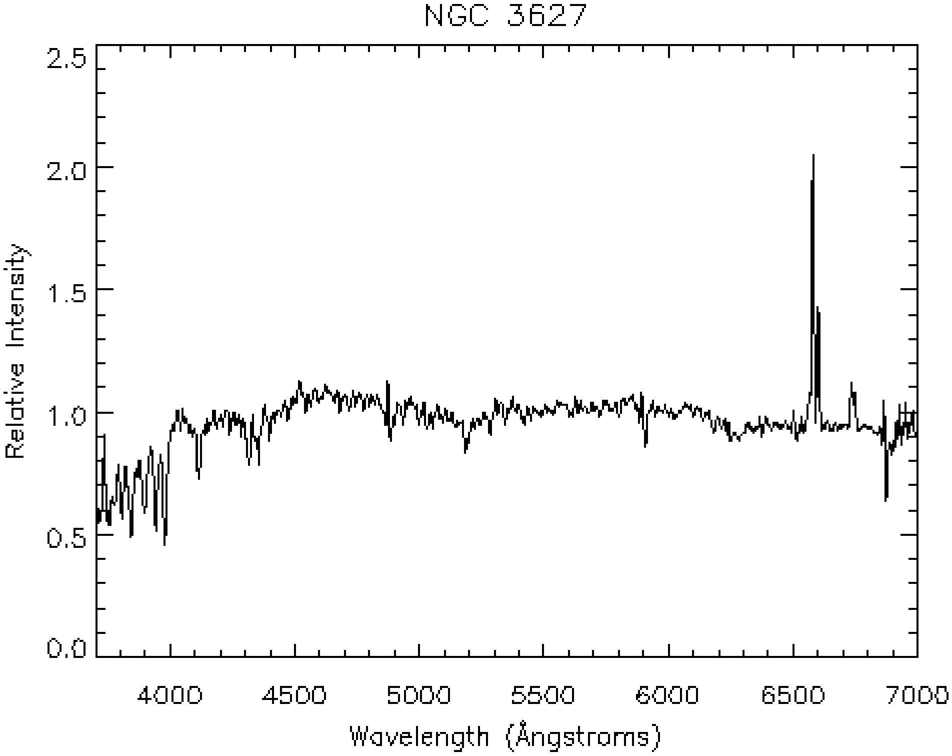
5000
5500
6000
6500
7000




NGC 3627
Relative Intensity
0.0
0.5
1.0
1.5
2.0
2.5
Wavelength (Ångstroms)
4000
4500
5000
5500
6000
6500
7000

Results:

1. Hubble's constant = 71.785
2. Age of the Universe = 13.621 Billion Years.

Conclusion:

Clear Redshift is observed in spectral lines of galaxies. The only reasonable explanation of this is that all galaxies are receding away from us. The farther a galaxy is, the faster it was moving away.

From this we can conclude that, Universe as a whole is expanding, now a day.

Wherever you look, distant stars are moving rapidly away from us. In other words, the universe is expanding. This means that at earlier times objects would have been closer together. In fact, it seemed that there was a time about twenty million years ago when they were all at exactly the same place.

This discovery finally bought the question of the beginning of the universe into realm of science. Hubble's observations suggested that there was a time called the big bang when the universe was infinitesimally small and, therefore, infinitely dense. If there were events earlier than this time, then they could not affect what happens at the present time. Their existence can be ignored because it would have no observational consequences.

One may say that time had a beginning at the big bang in the sense that earlier times simply could not be defined.

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A

Project On

Study on negative index of metamaterials

(THEORETICAL)

Submitted By

1. Zohra Noorain Aejaz

2. Priti Laxmikant

3. Shridhar Beerappa

4. Shivaprasad Kupendra

5. Jyoti Dattappa

Under the guidance of

Proff. C S Munnolli

Department of physics

A. V. Patil Arts, science and Commerce degree college Aland

HKE Society's

A V PATIL ARTS, SCIENCE & COMMERCE COLLEGE ALAND

2021-22

CERTIFICATION OF COMPLETION

This is to certify that the project report on **"Study on negative index of metamaterials"**

HKE'S A V PATIL ARTS, SCIENCE & COMMERCE COLLEGE ALAND is carried out under the guidance of Proff. C S Munnolli Lecturer and is submitted to the Department of Physics during the academic year 2021-22

Submitted by

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2. **Priti Laxmikant**
3. **Shridhar Beerappa**
4. **Shivaprasad Kupendra**
5. **Jyoti Dattappa**

Guide

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HKES's A.V. Patil Degree College
ALAND

Principal

PRINCIPAL
H.K.E. Society's A.V. Patil Degree
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We owe to him for giving me such an interesting topic and the way to see through the things it works in nature.

Also our genuine thanks to Mr.Sagar Kulkarni and Mr. Viquar Ahmed for their kind support, my parents and friends who give me the spirit to accomplish what had assigned to us.

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Abstract

Negative index metamaterials (NIMS) are materials having negative refractive index because of simultaneous negative permittivity and permeability in a certain frequency range. Theoretical concept of NIMS was introduced by a Russian Physicist; Vaselago in 1967. But the natural incurrence of these materials hampered the advancement in this field for a long time. It was around 2000 that the design of NIMS became possible using structured materials (composite of split ring resonator and thin wire), whose inhomogeneities are on length scales much smaller than the wavelength of radiation. A variety of resonance can be performed to obtain these materials in the various frequency ranges of the electromagnetic spectrum. Up till now we are able to work in the microwave and terahertz frequencies. NIMS are found to show phenomena like reversed Snell's law, reversed Doppler Effect, obtuse angle Cherenkov radiation etc. The most striking application of NIMS is the perfect lens wherein sub wavelength image resolution is made possible using a single slab of NIM as a lens. The possibility of soliton formation in NIMs is also the current area of study. Experimental work is still on the pace to fabricate metamaterials which can work in higher frequencies like visible range. In this project, attempt is made to understand NIMs in detail.

1. *INTRODUCTION*

In optics, the refractive index (R.I.) of a material is conventionally taken to be a measure of optical density and is defined as

$$n = \frac{c}{v},$$

where c is the velocity of light in vacuum and v is the velocity of the electromagnetic plane wave in the medium.

From Maxwell's equation, we have

$$n^2 = \sqrt{\varepsilon\mu}$$

This implies $$n = \pm\sqrt{\varepsilon\mu},$$

where ε is the relative permittivity and μ is the relative permeability of the medium.

Naturally occurring materials has n= +ve value and they are termed as Positive Index Materials (PIM) or Right Handed Materials (RHM). But for the case where ε<0 and μ<0, Vaselago in 1967 [1] proposed that

$$n = \sqrt{(-\varepsilon)(-\mu)}$$

$$= \sqrt{\varepsilon e^{i\Pi}}\sqrt{\mu e^{i\Pi}}$$

$$= -\sqrt{\varepsilon\mu}$$

Such materials with simultaneous negative values of both ε and μ so that we can have refractive index negative at overlapping frequencies are known as Negative Index Metamaterials (NIMs) also known as Negative Refractive Material (NRM) or Left Handed Materials (LHM). Since ε and μ are dispersive it is necessary to take into account that n depends on frequency otherwise the energy of the field given by

$$W = \varepsilon E^2 + \mu H^2,$$

will be negative when ε and μ are negative , which is impossible. When frequency dispersion exists the energy W must be given in a different manner:

$$W = \frac{\partial(\varepsilon\omega)}{\partial\omega} E^2 + \frac{\partial(\mu\omega)}{\partial\omega} H^2,$$

which is positive for a very broad class of dispersion equations for ε (ω) and μ(ω) [2].

All causal materials are dispersive which means ε and μ are complex functions of the frequency. They are negative below the plasma frequency,

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2},$$

and

$$\mu(\omega) = 1 - \frac{\omega_m^2}{\omega^2},$$

where ω_p and ω_m are the electric and magnetic plasma frequency. However the approach towards absorptive resonances at lower frequencies increases the dissipation and hence their complex nature. So far there is no such material found in nature but its artificial fabrication is possible. Such materials are found to exhibit various strange phenomena. The interest in this field increases due to the possibility of superlens production using NIMs. It is also found that soliton can be formed when electromagnetic waves propagate through NIMs which will be a boon in the field of communication.

1.1. *CLASSIFICATION OF ELECTROMAGNETIC MATERIALS*

The electromagnetic (EM) response of a medium is determined by the values of ε and μ of that medium. Based on the relative signs of these two, the EM materials can be classified into four types in shown in figure (1) below.

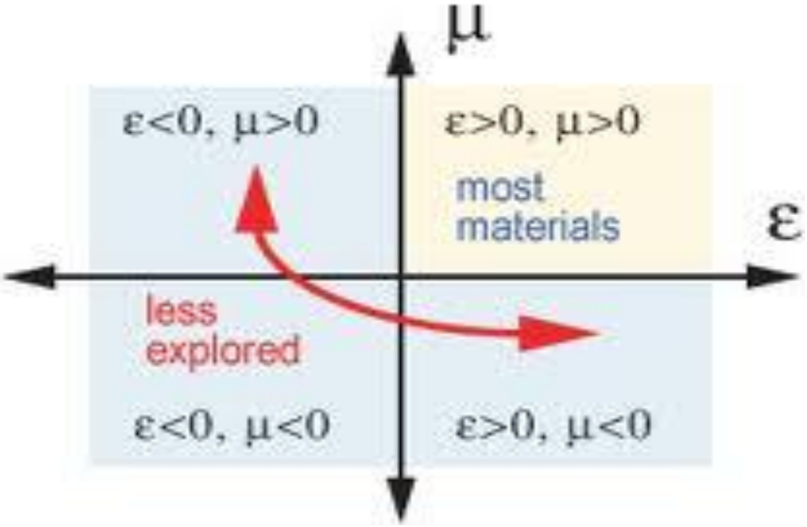

Fig (1): Re (ϵ) vs Re (μ) plane classifying em materials.

First quadrant- This corresponds to the normal material with ε>0 and μ>0. In such material we have propagating waves.

Second quadrant- ε <0 and μ>0 is the electric plasma material and here we get evanescent decaying waves. This material can support a host of resonant states localized at the surface known as surface plasmons.

Third quadrant- ε<0 and μ< 0 is the artificial NRM where we obtain propagating waves.

Fourth quadrant- ε >0 and μ <0 .This is the magnetic plasma material in which evanescent waves are obtained and can also support surface plasmons.

Our aim is to make a material of third quadrant using a composite of second and fourth quadrant materials over a common frequency range.

1.2 *PROPERTIES OF NIMS WHICH MAKE THEM DIFFERENT FROM NORMAL MATERIAL*

To study the electrodynamics of NIMs [1] which make them counter intuitive w.r.t the normal material let us consider the Maxwell's curl equations,

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{d\vec{B}}{dt} \qquad (1.2a)$$

$$\vec{\nabla} \times \vec{\mathrm{H}} = \frac{1}{c}\frac{d\vec{D}}{dt} \qquad (1.2b)$$

(a) For a plane harmonic wave $e^{i(\boldsymbol{k}.\boldsymbol{r}-\omega t)}$ (1.2a) and (1.2b) reduce to

$$\vec{k} \times \vec{E} = \omega\mu\mu_0\vec{H}$$

$$\vec{k} \times \vec{H} = -\omega\epsilon\epsilon_0\vec{E}$$

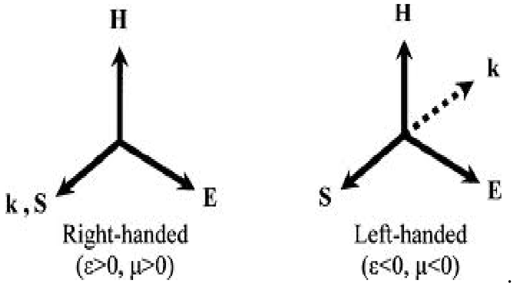

Fig (2): right handed and left handed triad.

For a medium with negative real parts of ϵ and μ with imaginary parts negligibly small $\overrightarrow{E}$, $\vec{k}$ and $\vec{H}$ form a left-handed triple of vectors whereas they form a right handed triple in normal materials.

(b) Poynting vector, $\vec{S} = \vec{E} \times \vec{H}$ and

$$\vec{k} = \frac{n\omega}{c}\hat{n} ,$$

where $\hat{n}$ is a unit vector along $\vec{E} \times \overrightarrow{H}$. This shows that $\overrightarrow{S}$ and $\overrightarrow{k}$ are parallel for n > 0 and antiparallel for n < 0. Thus in NIMs, waves propagate in a reverse phase. We also know that the phase velocity of the wave coincides with the direction of $\vec{k}$ and group velocity with $\vec{S}$. Therefore $\overrightarrow{S}$ is antiparallel to phase velocity in NIMs which means the phase wavefronts move backward.

(c) Group velocity, v_g is opposite to phase velocity, v_p in LHM.

Since
$$v_p = \frac{\omega}{k}\hat{k}$$
$$= \frac{\omega c}{n\omega}\hat{k}$$

Phase velocity in LHM is opposite to that in RHM. In linear, isotropic non dissipative media, group velocity is equal to the energy flow velocity associated with $\vec{S}$ which does not depend on material properties.

Hence for LHM phase velocity and group velocity are of opposite sign and wavefront travels towards the source.

1.3. *FABRICATION OF NIMs*

In nature we do not find any material exhibiting negative refraction at any frequency. But the theoretical implications suggest various useful applications for such material. So NIMs are artificially fabricated for the first time in the year 2000 and the fabrication consists of making a composite of an array of thin wires showing negative permittivity and split ring resonators (SRR) with negative permeability such that it has artificially designed arrays of LC oscillators mounted on electronic circuit plates capable to interact with em fields with frequency around 10 GHz. Graphing the general dispersive curve for SRRs, a region of propagation occurs from 0 up to a lower band edge followed by a gap and then an upper passband. When wires are symmetrically added between the splits rings a passband occurs within the forbidden gap.

Fig (3): composite of thin wire and SRRs

Most materials exhibiting a good electrical response can be found at almost any frequency from radiofrequency to ultraviolet frequencies but the magnetic response of most materials is limited to low microwave frequency as the magnetic polarization usually results from either unpaired electron spins or orbital electron currents. Therefore the collective excitations of these usually tend to occur at low frequency (microwave).

Let us study these components separately.

Thin wire medium: Mesh wire structures which consist of composites of randomly oriented long conducting fibers have been known to exhibit very high values of permittivity even at low concentration. Effective medium theories describe these systems when the wavelength of the incident radiation is much larger than the intrinsic length scales of the structure. However the radiation probes only the end surfaces of the metallic structures and hence it is hard to make it penetrate well into the bulk of the structure for the appearance of three dimensional effective medium to hold true in many cases. Pendry et al [3] and Sievenpiper [4] independently demonstrated that metallic wire-mesh structures have a low frequency stop band from zero frequency up to a cut off frequency which they attributed to the motion of electrons in the metal wires and therefore we can obtain a negative dielectric at low frequency.

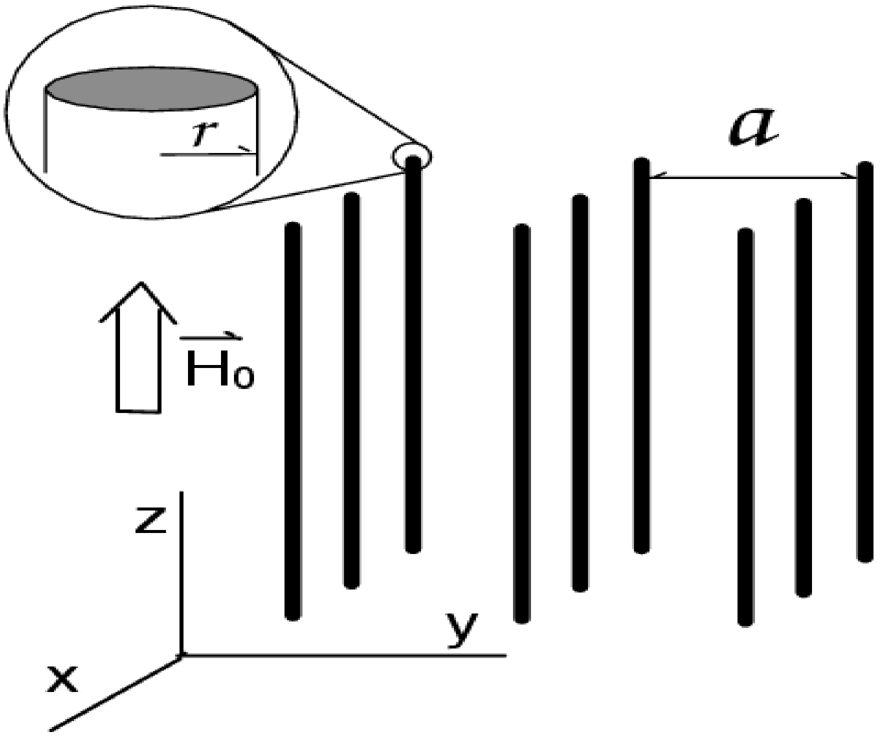

Fig (4): wire-mesh metallic structure as effective negative permittivity medium

By inherent property of thin wire medium, it has negative permittivity at frequencies below plasma frequency. Due to spatial confinement of the electrons to thin wires, the effective electron concentration in the volume of the structure is reduced which also decreases the plasma frequency. Thus an array of thin metallic wires by virtue of its macroscopic plasma like behaviour produces an effectively negative permittivity at microwave frequency.

For obtaining negative permittivity we exclude sphere and disc type media since the finite dimensions of these conducting inclusions transverse to applied field make the effective medium a diamagnetic response.

DISPERSION RELATION FOR PERMITIVITY ε (ω)

The dispersion relation is obtained using Drude-Lorentz model [5] as discussed below. The free electrons of conductors are considered to as negatively charged plasma. The long wavelength dielectric response ϵ (ω) of an electron gas is obtained from the equation of motion in an electric field.

$$m\ \ddot{x} = -e\ E$$

If x and E have time dependence e^{-iwt}, then

$$-\omega^2 m\ x = -e\ E$$

$$\Rightarrow \quad x = \frac{eE}{m\omega^2}$$

The dipole moment of one electron is

$$-e\ x = -\frac{e^2}{m\omega^2} E\ .$$

The polarization defined as dipole moment per unit volume is

$$P = -\ ne\ x$$

$$= n \frac{e^2}{m\omega^2} E,$$

where n is electron concentration.

Since we know that, $\varepsilon\ (\omega) = \frac{D(\omega)}{E(\omega)} = 1 + 4\text{л} \frac{D(\omega)}{E(\omega)}$

$$\Rightarrow \varepsilon\ (\omega) = 1 + 4л \left(-\frac{ne^2}{m\omega^2}\right) \text{ (c.g.s)}$$

or $$\varepsilon(\omega) = 1 - \frac{e^2}{\epsilon_0} \frac{n}{m\omega^2} = 1 - \frac{\omega_p^2}{\omega^2} \qquad (1)$$

If we consider the dissipation into account the relation is

$$\varepsilon\ (\omega) = 1 - \frac{e^2}{\epsilon_0} \frac{n}{\omega(\omega + i\gamma)}\ ;$$

γ is the damping or dissipation factor.

This is the dispersion relation for $\varepsilon(\omega)$ and it is negative for $\omega < \omega_p$ (plasma frequency).

Plasma frequency is defined by the relation

$$\omega_p^2 = ne^2/\varepsilon_0 m \quad \text{or} \quad \omega_p^2 = 4\pi ne^2/m$$

Split Ring Resonator (SRR): It consists of two rings with oppositely oriented splits. The splits in the rings are responsible for resonance at wavelengths much larger than the diameter of the rings [6]. The second split is oppositely oriented to generate a large capacitance at the small gap. With a single split a large electric dipole moment will be generated across the capacitive gap and this could well dominate over the weaker magnetic dipole moment generated in the ring. When there are two oppositely oriented splits, the dipole moment across opposite splits cancel each other and one only gets weak electric quadrupole moment whose effects can be dominated by the magnetic dipole moment. The periodic array of SRRs allows material to behave as a medium with effective μ at resonance since the incident wavelength cannot sense each individual unit. What we get is a response average over all the units. It works on the principle that a magnetic flux penetrating the metal rings will induce rotating currents in the rings which produce their own flux enhancing or opposing the incident field depending on the spin. This field pattern is dipolar.

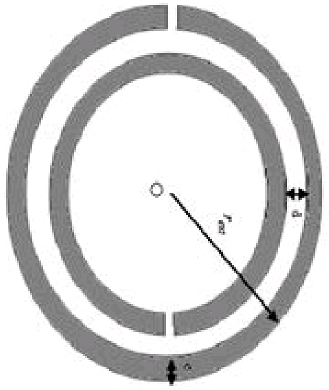

Fig (5): SRR

The magnetic flux produced can be understood as magnetic response. In other word, we can say that when an alternating magnetic field is applied perpendicular to the plane of split ring resonator, it behaves like a magnetic driven LC circuit exhibiting a resonance response at frequency Ω_m associated with the resonant circular currents in the SRR.

Where, $\Omega_m = \frac{1}{LC}$

This resonant circular currents give rise to a resonant magnetic dipole moment thereby we can recognize a SRRs system as a resonant effective permeability.

DISPERSION RELATION FOR PERMEABILITY μ (ω)

Consider the SRRs to be placed in a square lattice of lattice constant, a. In a SRR assuming the gap to be very small compared with the radius (r) and that the capacitance due to the large gaps in any single ring is negligible, we balance emf around the circuit with the ohmic drop in potential (Lenz Law).

By Lenz Law, $-\frac{d\phi}{dt} = -\frac{d}{dt}\int B.ds = \int E.dl - \frac{j}{i\omega c}$

.

$$\Rightarrow i\omega\mu_0 л\, r^2\left(H_0 + j - \frac{лr^2}{a^2} j\right) = 2л\, r\, \rho\, j - \frac{j}{i\omega c},$$

Here we use, $B = \mu_0 H$ and $j = \sigma E$

$H = H_0 + j - \frac{лr^2}{a^2} j$, is the axial magnetic field inside the SRR

H_0 = applied magnetic field, j = induced current per unit length.

And third term is the depolarizing field due the induced current. ρ is the resistance per unit length.

$C = \frac{\epsilon_0}{3d}\varepsilon\pi r$ is the effective capacitance with ε as the relative dielectric permittivity of the material in the gap, d. Now for a homogeneous system of SRRs the effective magnetic field $H_{eff} = H_0 - \frac{лr^2}{a^2} j$.

Then $B_{eff} = \mu_0 H_0$

$$\mu_{eff} = \frac{B_{eff}}{\mu_{0H_{eff}}}$$

Solving we get, $$\mu_{eff} = 1 - \frac{{\text{л}r^2}/{a^2}}{1-\left({3d}/{\mu_0\epsilon_0\epsilon\text{л}^2 r^3\omega^2}\right)+ i({2\rho}/{\mu_0\omega\, r})}$$

$$= 1 + \frac{f\omega^2}{\omega_0^2-\omega^2-i\Gamma\omega}$$

Here $\omega_0 = \sqrt{\left(\frac{3d}{\mu_0\epsilon_0\epsilon\text{л}^2 r^3}\right)}$ is the resonant frequency and f= $\frac{\text{л}r^2}{a^2}$ is the filling fraction of the material. For frequencies larger than ω_0, the response is out of phase with the driving magnetic field and μ_{eff} is negative upto the magnetic plasma frequency given by

$$\omega_m = \sqrt{\frac{3d}{(1-f)\mu_0\epsilon_0\epsilon\text{л}^2 r^3}} \qquad (2)$$

TO SHOW NIMs WORK IN MICROWAVE REGION

Using the dispersion relation,

$$\epsilon = 1 - \frac{\omega_p^2}{\omega(\omega+i\gamma)}$$

Where, γ = damping factor and ω_p = plasma frequency.

For small damping $\gamma = 0$, from (1) we get

$$\epsilon = 1 - \frac{\omega_p^2}{\omega^2} \text{ and } \omega_p = \frac{ne^2}{m\epsilon_0}$$

For density of plasma (n) in the wires ~10^{17} $/m^3$

And putting the values of m= mass of electron, e = 1.6×10^{-19} C

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ N } m^2\, C^{-2}$$

We get $\omega_p \sim 10^{10}\ s^{-1}$ which corresponds to wavelength $\sim 10^{-2}$ m i.e in the microwave region.

For ϵ to be negative ω < ω_p and it is possible in the microwave region.

Similarly in (2) if we put the values of r = 1.5 mm, a = 5 mm, d = 0.2 mm we have a resonant frequency in the microwave region where μ is negative.

1.4. *PHENOMENA EXHIBITED BY NIMs*

Some of the strange phenomena exhibited by NIMs are discussed as below.

(a) Reversed Snell's Law

According to Snell's Law,

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Where $n_1 and\, n_2$ are the refractive indices (R.I) of the rarer medium and denser medium and θ_1, θ_2 are the angles of incidence and refraction respectively. When both n_1 *and* n_2 are positive refracted ray is on the opposite side of the normal while it is refracted on the same side of normal when $n_1 > 0$ but $n_2 < 0$ as shown in figure 6 below.

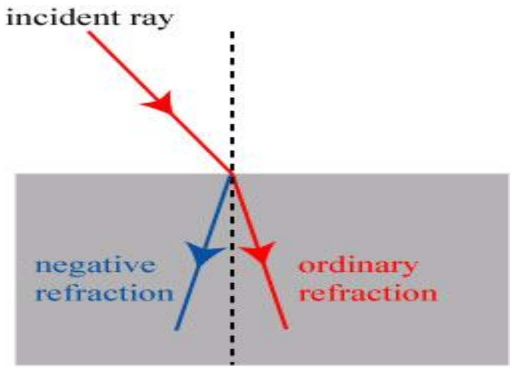

Fig (6): diagram shows positive and negative refraction

When $n_1 > 0$ but $n_2 < 0$ modified Snell's Law becomes

$$n_1 \sin\theta_1 = -\sin\theta_2$$

$$n_1 \sin\theta_1 = \sin(-\theta_2)\ ,$$

and hence the negative refraction.

(b) Reversed Doppler Effect

In Doppler Effect the frequency of a source increases or decreases when a detector is moving towards or away from it. But the thing is reversed in case of Reversed Doppler Effect.

Suppose if a source emits radiation at frequency ω and a detector is moving w.r.t source at velocity v, then the frequency received by the detector is given by

$$\omega' = \gamma\ (\omega + \vec{k}.\vec{v})$$
$$= \frac{1}{\sqrt{c^2-v^2}}\ \omega\ (1 + \frac{nv}{c}) \text{ where } |\vec{k}| = \frac{n\omega}{c}$$

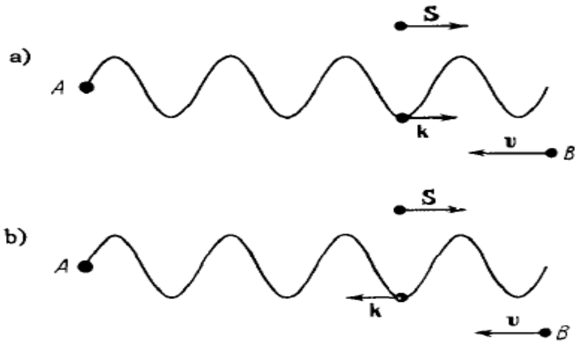

Fig (7): Doppler Effect in a right-handed substance; b) Doppler Effect in a left-handed substance. The letter A represents the source of the radiation, the letter B the receiver.

If n = +1,

$$\omega' = \omega\sqrt{\frac{c+v}{c-v}}\ .$$

But for NRM, $\vec{k}$ has negative sign since n (refractive index) is negative and

$$\omega' = \omega\sqrt{\frac{c-v}{c+v}}$$

Thus the frequency received by the detector will increase as the source is receding from it and vice versa.

(c) Obtuse angled Cherenkov's radiation

Cherenkov radiation is the cone of electromagnetic radiation when a charge particle such as electron passes through a dielectric medium at a speed greater than the phase velocity of light that medium. The charged particles polarize the molecules of the medium which then turn back rapidly to their ground state emitting radiation in the process.

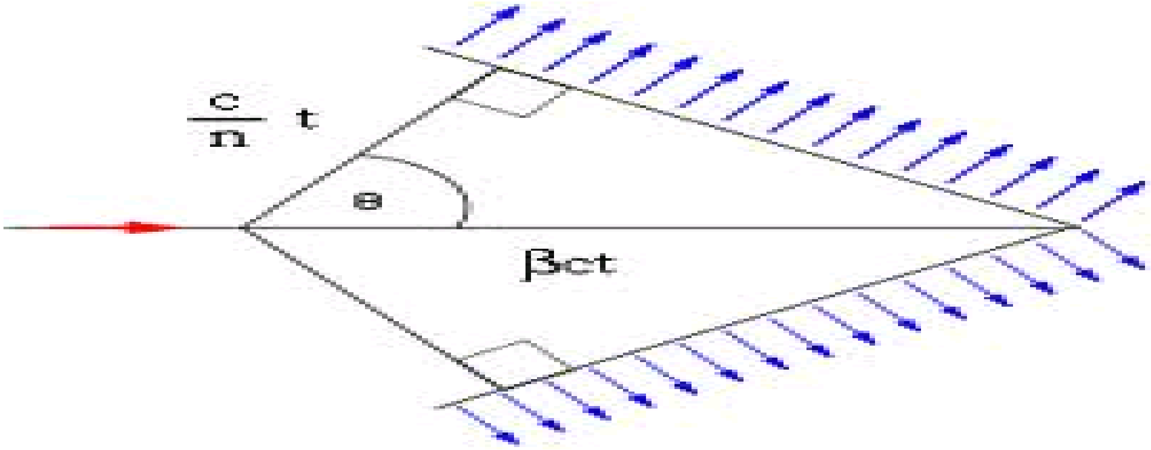

Fig (8): Cherenkov radiation.

Suppose at time t = 0, charge particle is situated at left hand corner of the diagram and traverses to the right corner with velocity phase velocity equal to βc in time t as shown in fig. 8 above. Distance traversed will be equal to βc t. If n is the R.I of the medium the cone will have traversed a distance $\frac{c}{n}$ t. Hence the acute angle of this cone is given by,

$$\cos\theta = \frac{c/n}{\beta c} = \frac{1}{\beta n}$$

For positive value of n, θ is acute while for the case of NRM n is negative and hence we will have radiation from a cone of obtuse angle.

1.5. *THEORY OF DIFRACTION LIMIT*

Consider an object and a lens placed along the z-axis so that the rays from the object are travelling along the z direction. The field emanating from the object can be written in terms of superposition of plane waves.

$$E(x, y, z, t) = \sum_{k_x k_y} A(k_x, k_y) e^{i(k_z z + k_y y + k_x x - \omega t)}$$

Where, $$k_z = \sqrt{\frac{\omega^2}{c^2} - (k_x^2 + k_y^2)}$$

Only positive square root is taken as the energy is going in the +z direction. All the components of the angular spectrum of the image for which k_z is real, are transmitted and refocused by an ordinary lens.

However if $k_x^2 + k_y^2 > \frac{\omega^2}{c^2}$ (higher resolution case), then k_z becomes imaginary and the wave is an evanescent wave whose amplitude decays as the wave propagate along the z- axis. The result is the loss of high frequency components of the wave which contain information about the high frequency features of the object being imaged. The highest resolution that can be obtained in a conventional lens is

$$k_{max} \simeq \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

$$\therefore x_{min} \simeq \lambda$$

If the lens is placed at a distance larger than the operating wavelength, λ then k_z component will not be seen. In Pendry's Perfect lens, the transport of energy in the +z direction requires k_z to have opposite sign.

$$k_z = -\sqrt{\frac{\omega^2}{c^2} - (k_x^2 + k_y^2)}$$

For large angular frequencies, the evanescent wave grows so with proper lens thickness, all components of the angular spectrum can be transmitted through the lens undistorted. Thus the perfect lens is capable of capturing the near field components.

2. *APPLICATIONS*

2.1. *PERFECT LENS*

We consider a Vaselago's perfect lens [1] which consists of a slab of NRM with $\epsilon = -1$ and $\mu = -1$ capable of focusing both the propagating and evanescent waves emitted by an object.

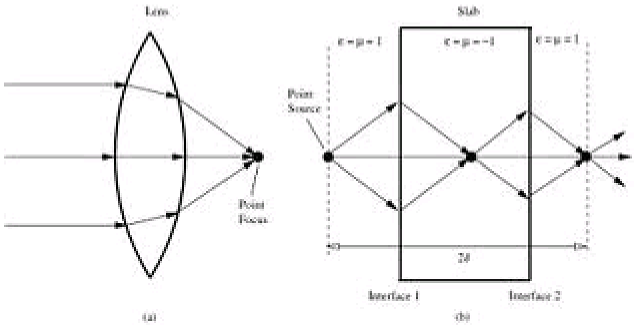

Fig 9 (a): Conventional lens Fig 9 (b): perfect lens

When an object is placed in front of an material with $n = -1$, the waves are refracted so that they focus once inside the lens and once outside it. Such refraction allows for sub-wavelength resolution. Hence a perfect lens allows the near field rays to occur once within the lens and once outside enabling sub-wavelength imaging.

Inside the perfect lens, the amplification of the evanescent waves take place by producing excited states at the NRM surfaces. For this the surface current matches the evanescent waves from the object. We can mathematically show it using Pendry's Proposal.

Consider a Vaselago's lens consisting of a slab of thickness d with $\epsilon = -1$ and $\mu = -1$ surrounded by vacuum as shown in figure above. Source is at z = 0 (object plane). We are to calculate the fields at z = 2d (image plane).

The transmission and reflection coefficients at the interfaces are

$$T=\frac{t_{21}t_{32}e^{ik_{z2}d}}{1-r_{12}r_{21}e^{2ik_{z2}d}}$$

$$R=\frac{r_{21}+r_{32}e^{2ik_{z2}d}}{1-r_{12}r_{21}e^{2ik_{z2}d}}$$

When NRM has $\epsilon_{-}=-\epsilon_{+}=-1,\ \mu_{-}=\mu_{+}=-1$, we obtain trivially for propagating waves $k_{z2}=-k_{z1}$, $t_{jk}=1$ and $r_{jk}=0$ due to matched impedance.

$$\lim_{\substack{\epsilon_{-}\to-1\\ \mu_{-}\to-1}} T = e^{-ik_{z1}d}$$

And $$\lim_{\substack{\epsilon_{-}\to-1\\ \mu_{-}\to-1}} R = 0$$

This clearly shows that the total phase change for propagation from the object plane to the image plane is zero.

For evanescent waves with $$k_{z1}=i\sqrt{k_x^2+k_y^2}-\epsilon_{+}\mu_{+}\frac{\omega^2}{c^2}$$

$$=ik_z$$

Then $k_{z2}=k_{z1}$ and the partial coefficients $t_{jk}\ and\ r_{jk}$ diverge.

However the transmission and the reflection coefficients of the slab are still well defined in this limit.

$$\lim_{\substack{\epsilon_{-}\to-1\\ \mu_{-}\to-1}} T = e^{k_z d}$$

$$\lim_{\substack{\epsilon_{-}\to-1\\ \mu_{-}\to-1}} R = 0$$

i.e the slab actually increases exponentially the amplitude of the evanescent wave at the same rate by which it decays in free space.

Differences between conventional and perfect lens

Conventional lens	Perfect lens
(a) Its resolution is limited by the diffraction limit.	(a)Its resolution is not subjected to the diffraction limit.
(b) It focuses only the propagating wave of the electromagnetic radiation.	(b) It can focus both the propagating and evanescent waves of the electromagnetic radiation.
(c) A convex lens shows a converging nature and a concave lens a diverging one.	(c) A concave lens shows a converging nature and a convex lens a diverging one.

2.2. *CLOAKING*

The phenomenon of concealing an object from view is called cloaking. The principle of cloaking was first achieved in the microwave frequency on Oct 19, **2006**. An object is made invisible by covering it with a metamaterial cloak due to its ability to deflect the electromagnetic radiation. The radiation flows around the object as if nothing were there at all. We know that the bending of light is determined by refractive index. Metamaterials have a gradient in refractive index since it is inhomogeneous. The existence of this gradient in NIMs makes possible the creation of cloaking devices. Moreover the bending of light can be explained by Transformation Optics.

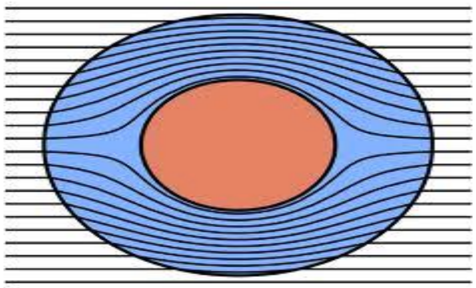

Fig 10 (a): diagram showing bending of light

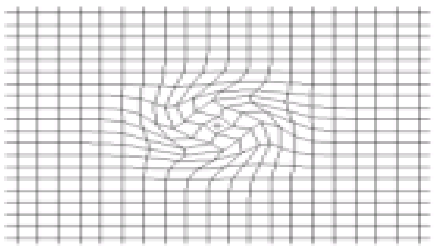

Fig 10 (b): twisting space coordinate

By transformation optics [7], if we twist the optical metamaterial it will affect its space into new coordinates. Maxwell's equations remain intact but change in the values of ε and μ by a common factor takes place and hence is the change in the value of refractive index. Then the light will travel in real space with a curved fashion in the twisted space.

Before conclusion we would like to point out an interesting aspect of EM propagation through NIMs. There is possibility of soliton formation in NIMs when EM waves propagate through it. So soliton formation in NIMs is an area of current research. Solitons [8] or solitary waves are the waves which are of localized shape and continue to travel with constant velocity for a long time without dissipating their energy. It is produced when non-linearity is cancelled out by dispersion. In a metamaterial, most of the electrostatic energy of the capacitor is located in the gap between the two rings which results in an enormously enhanced energy density making it a non-linear material and hence possibility for soliton formation. Such waves have been seen in many natural systems such as water waves [9], non-linear optics [10], BEC systems [11] and in quantum field theory (in 1, 2, 3 dimensions) [12]. Dispersive coefficient GHNLSE is a prototype equation which describes the propagation of EM waves in NIMs or non-linear optical fiber. Study of localized solutions in such media is current research interest.

3. *CONCLUSION*

In this project, the physics of the NIMs are studied with the values of ε and μ negative at overlapping frequencies when the wavelength of the incident radiation is very large compared to the inhomogeneities of the length scale. The most common method of fabrication is found to be the composite of thin wire medium and SRRs. The various properties of such materials have been discussed and the various strange phenomena exhibited by them are due to the fact that the propagation vector moves towards the source. It is also shown that the diffraction limit is being removed with perfect lens made of a slab of such material and the possibility to cloak an object in the microwave region. Soliton formation in NIMs will also be an effective tool for communication since it is it is non-dissipating. The future prospect is to make NIMs capable to work in the visible region which will be possible if we have the advanced technology to scale down the size of the unit structure.

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